

P O L Y K A R P K U S C H

The magnetic moment of the electron

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I must tell you, and with considerable regret, that I am not a theoretical physicist. A penetrating analysis of the part that the discovery and measurement of the anomalous magnetic moment of the electron has played in the development of certain aspects of contemporary theoretical physics must be left to the group of men who have in recent years devised the theoretical structure of quantum electrodynamics. My role has been that of an experimental physicist who, by observation and measurement of the properties and operation of the physical world, supplies the data that may lead to the formulation of conceptual structures. The consistency of the consequences of a conceptual structure with the data of physical experiment determines the validity of that structure as a description of the physical universe. Our early predecessors observed Nature as she displayed herself to them. As knowledge of the world increased, however, it was not sufficient to observe only the most apparent aspects of Nature to discover her more subtle properties; rather, it was necessary to interrogate Nature and often to compel Nature, by various devices, to yield an answer as to her functioning. It is precisely the role of the experimental physicist to arrange devices and procedures that will compel Nature to make a quantitative statement of her properties and behavior. It is in this spirit that I propose to discuss my participation in a sequence of earlier experiments that made possible the precision determination of the magnetic moment of the electron. I will then discuss the experiments themselves which have yielded our present knowledge of the magnetic properties of the electron.

Research with atomic and molecular beams has had a long and fruitful record in the history of the growth of our present knowledge of matter. The experiments that I shall discuss are some in which the method of atomic and molecular beams is used essentially as a spectroscopic device for the observation of spectral lines in the range of frequencies within which power may be generated by electronic means. The general principles of radiofrequency spectroscopy by the method of molecular beams were first described by Rabi¹ and a group of his co-workers of which I was fortunate to be a mem-

ber. It is here sufficient to say that a transition between energy levels may be observed through the circumstance that the magnetic moment of an atom or molecule may be changed in a transition. The method is characterized by a very high potential resolution, and in many observations of the frequency of a line, an accuracy of better than one part in a million has been achieved. It is of particular value as a tool in the investigation of the details of interactions within atoms and molecules because small interactions appear as first-order effects rather than as small superpositions on the relatively enormous energies that characterize optical spectra.

The fact that the electron has a spin of one half and a magnetic moment at least approximately equal to one Bohr magneton has long been recognized. Uhlenbeck and Goudsmit² first postulated these properties of the electron to explain the fine structure in atomic spectra and what has been called the anomalous Zeeman effect. An enormous body of evidence has given an ever-increasing support to these postulates. The relativistic Dirac theory of the electron assumed a particle that was endowed with the properties of mass and charge. The spin and magnetic moment postulated by Uhlenbeck and Goudsmit were then found to be a consequence of the relativistic invariance of the Dirac equation. Indeed, one of the great triumphs of the Dirac electron theory was the prediction of these postulated electron properties. The spin and moment of the electron were thus removed from the realm of *ad hoc* assumptions, justified by experimental evidence, to the realm of an integral part of quantum theory. The Dirac electron theory did not, however, consider the interaction of the quantized electromagnetic field with the electron.

I shall talk of the measurement of the g value rather than the magnetic moment of the electron. The g value is, as usual, the negative ratio of the magnetic moment in terms of the Bohr magneton μ_0 and the angular momentum in units of $h/2\pi$. Since, in all cases here under discussion, the angular momentum of the system is known, the moment can immediately be obtained from the g value. The most elementary of the g values, g_l , is that associated with the orbital motion of the electron. Its value is 1 within small and calculable corrections. The electron also has a magnetic moment by virtue of its angular momentum about a spin axis. The g value associated with the spin, g_s , is the quantity here under investigation; a value of 2 was obtained for it in the Dirac electron theory. Now the electrons in an atom have both spin and orbital angular momentum. To the total electronic angular momentum J , we assign the g value g . The atom contains a nucleus that may have a nuclear angular momentum and hence a nuclear magnetic moment.

The nuclear g value, g_N , is designated as g_p in the special case when the nucleus is a proton. To the total angular momentum of the atom we assign the g value g_F .

The earliest measurements by the molecular beams magnetic resonance method were undertaken by a group¹ of which I was a member working in Professor Rabi's laboratory and under his direction. The measurements consisted of the determination of nuclear g values by the observation in a molecule of the nuclear resonance frequency in a classically determined magnetic field. Even in the great national laboratories dedicated to the maintenance of physical standards, a precision of only about one part in forty thousand has been achieved³ in the measurement of a field. In a well-equipped laboratory that, however, lacks the equipment and tradition of meticulous intercomparison of electrical standards, a precision of perhaps one quarter percent may be achieved in the determination of the magnitude of a field. While ratios of nuclear g values may be found without an explicit knowledge of the field, the accuracy of the determination of a nuclear moment in terms of the Bohr or nuclear magneton is limited by the uncertainty in the measurement of a field as well as by the uncertainty in a prior measurement of the Bohr or nuclear magneton. Thus, the desirability of a direct measurement of a nuclear g value in terms of the Bohr magneton is apparent.

The molecular beam magnetic resonance method was originally applied to the determination of the nuclear g values in molecules that did not have a net electronic orbital or spin angular momentum. It is, however, possible to apply the same experimental techniques to an investigation of the hyperfine structure of atoms. If we observe transitions between the various F levels at zero or very low magnetic field, the hyperfine structure separation may readily be found. At a higher magnetic field the observation of the frequency of transition between magnetic sub-levels yields again the zero-field h.f.s. (hyperfine structure) splitting, the quantity $g_J\mu_0H/h$ and the quantity $g_N\sqrt{g}$, though the latter quantity can be found with only limited precision. The group of which I was a member at Columbia made⁴ the first such studies on the commonly occurring isotopes of the alkali atoms and determined the magnetic hyperfine structure interaction constants of the alkali atoms. Extensive subsequent work in the observation of atomic h.f.s. has, of course, been done in many laboratories with results of great interest in the study of higher-order moments in nuclei and of the properties of radioactive nuclei. The alkali atoms were particularly adaptable to the original experimental work, first because the beams of the atoms may readily be produced and de-

tected and secondly, because they occur in ${}^2S_{1/2}$ states, almost wholly free of perturbation by other states.

The possibility of measuring the moment of a nucleus in terms of the Bohr magneton is a consequence of the possibility of observing both nuclear resonance in molecules with a frequency $g_N\mu_0H/h$ and transitions among the magnetic components of h.f.s. levels for which the dependence of frequency on field is of the order of $g_J\mu_0H/h$. Millman and I then addressed ourselves to the problem of measuring the moment of the proton in terms of the electron spin moment. The experimental problems were considerable and arose from three factors. The first of these was related to the fact that the effective moment of a molecule of zero electronic angular momentum is of the order of a nuclear magneton while that of an atom is of the order of a Bohr magneton. Deflecting fields that allow the observation of a change in trajectory of a molecule in which a hydrogen nucleus has undergone a transition will deflect an atom through unmanageably large excursions if the field is arbitrary. However, all atoms in which magnetic h.f.s. occurs and in which the spin of the nucleus is greater than $1/2$ have, in certain states, zero magnetic moments at definite values of the magnetic field which may be very high. Atoms in such states may thus traverse a carefully adjusted inhomogeneous field without catastrophic deflections, and a transition from such a state may at once be detected since the terminal state is generally characterized by a large magnetic moment. Thus, it is possible to choose the deflecting field in such a way that a change in the spin orientation of a nucleus in a molecule and a transition among the magnetic levels of the h.f.s. may both be detected. The second of our experimental problems was related to the production of a beam of molecules that contained hydrogen and simultaneously an alkali atom, requisite for the detection of beams by techniques then available to us. Since atomic lines were to be observed at the same field as nuclear resonance lines, the simultaneous production of a beam of alkali atoms was necessary. We used beams of sodium and potassium hydroxides evaporated from silver ovens and noted that at the temperatures required to generate a beam of the hydroxide, the reaction between an alkali halide and metallic calcium proceeded at such a rate that a convenient beam of atoms appeared. The third experimental problem was associated with the need of applying to the same circuit, in succession, two frequencies that differed by as much as a factor of seventy, and with the pre-war difficulty of generating r.f. (radio-frequency) power at high frequencies.

Extensive intercomparison of the frequencies of the resonance line of the

protons and of lines in the h.f.s. spectra of sodium, rubidium and caesium for each of which a prior determination of the interaction constants had been made, led to a determination of the ratio of the proton moment and the spin magnetic moment of the electron in the calibrating atoms which we, of course, assumed to be the Bohr magneton. The magnetic moment of the proton in terms of the nuclear magneton found on the basis of the assumption that the spin moment of the electron was indeed the Bohr magneton differed from the moment as determined from the measurement of a frequency in a classically determined field by about one tenth percent. When, at a much later date, it was found that the spin magnetic moment of the electron deviates from the Bohr magneton by the order of one tenth percent, the direction of the deviation in the older experiment was examined. It is perhaps a good commentary on the hazards of experimental physics that no significant effect had escaped us but that the error in the mutual inductance which we had used in the calibration of the magnetic field was of the order of two tenths rather than one tenth percent.

After the war, Nafe and Nelson⁶, working with Rabi, made the first of the measurements of the hyperfine structure splitting of hydrogen in the ground state. Now the h.f.s. of hydrogen may be calculated explicitly in terms of the magnetic moment of the proton, the spin magnetic moment of the electron and the electronic wave function at the nucleus. However, a discrepancy of about one quarter percent was noted between the observed and predicted magnitude of the h.f.s. splitting when the value of the proton moment found by Millman and me was used. The assumption that the spin moment of the electron is a Bohr magneton enters into the calculation twice, first as an intrinsic property of the electron and second in the calculation of the proton moment from the observed ratio of the proton moment and the spin moment of the electron. The discrepancy led Breit⁷ to suggest that the electron may possess an intrinsic magnetic moment greater than μ_0 by the order of $a\mu_0$, where a is the usual fine structure constant.

The question of the existence of an anomalous magnetic moment was then investigated in detail by Foley and me⁸. In this inquiry, as in all others conducted in the atomic and molecular beams laboratory at Columbia University, we profited by Rabi's advice. The procedure that we employed capitalizes on the fact that the g value associated with a state is a linear combination of the electronic orbital and spin g values, g_l and g_s and that this combination is different for different states. That is, there is a contribution to the total electronic magnetic moment of an atom both from the orbital mo-

tion of the electrons and from the spin of the electrons, and the contribution from each of these factors is dependent on the state of the atom. Since we considered only atoms with single electrons outside of closed shells, Russell-Saunders coupling is a good approximation and the coefficients that relate the various g values are known. The ratio of the g_i values of two atoms that occur in different spectroscopic states yields g_s/g_l to an accuracy limited by the precision of observation and the precision with which the coefficients relating the various g values are known.

The intercomparison of g values to obtain a value of g_s can only be made if atoms in several different spectroscopic states are available for observation. After our first investigation of the hyperfine structures of the alkali atoms, all in the ${}^2S_{1/2}$ state, Hardy and Millman⁹ studied the h.f.s. of indium in the ${}^2P_{1/2}$ state. Just after the war Becker and I¹⁰ determined the interaction constants that characterize the h.f.s. of both isotopes of gallium in the ${}^2P_{1/2}$ state. Gallium atoms in the excited and metastable ${}^2P_{3/2}$ state also occur in an atomic beam, and it was possible to determine the interaction constants for both isotopes in this state as well. We thus had available for study atoms in three different spectroscopic states.

In principle the determination of the ratio of two g_i values is simple. Suppose we observe transitions for which F is constant and m_f changes by ± 1 for two different atoms or for the same atom in two different states. To the extent to which strictly low field conditions prevail, all lines in a given F state have the same frequency and the ratio of the frequencies of such lines in two different states at a fixed field is simply the ratio of the g_f values. From this ratio, g_s/g_l for the electron may readily be derived with some additional knowledge of the properties of the nucleus in each atom. However, the h.f.s. splitting of atomic states is generally small (from 200 to 20,000 megacycles) and the energies of the levels are far from linearly dependent on magnetic field at usefully high fields. It is, nevertheless, possible to obtain expressions for the energies of all levels in the h.f.s. in terms of the zero field h.f.s. splitting, the ratio g_N/g_J and the quantity $g_J\mu_0 H/h$, or, where such expressions cannot be explicitly found, to determine the energies from the appropriate secular determinants. From the observed frequencies of appropriate lines and with a prior knowledge of the interaction constants which characterize an atom in the state in question it is then possible to determine $g_J\mu_0 H/h$. Measurement of this quantity at the same field for atoms in two different states yields, at once, the important ratio g_s/g_l . The determination is independent of a knowledge of the magnetic field and of any fundamental constants.

It is, perhaps, worthwhile to remark on some experimental details. The field in which the transition frequency was measured was so chosen that all the observed lines had a frequency of the order of 1 megacycle per gauss. To avoid excessive distortion of the lines due to inhomogeneity of the field, a great deal of adjustment of the field was required before the lines approximated in width their theoretical value.

Special arrangements were made to allow the rapid interchange of ovens so that lines of different atoms could be measured in rapid succession. A considerable number of oscillators was required so that several frequencies which differed by large factors could be applied to the r.f. circuits that induced the transitions. While the lines should, in principle, be measured at a fixed if unknown field, the actual measurements were made in a field that varied monotonically throughout a series of observations. The variation of field has the annoying effect of requiring a large body of data to establish the frequencies of two or more lines at a fixed field, but it also aids in avoiding repetitive errors that may occur when a reading of a fixed quantity is repeated.

Three intercomparisons of g_j values were made in these experiments. The results are given in Table 1.

Table 1. Observed ratios of atomic g values and the corresponding values of g_s/g_i .

<i>Comparison</i>	Nominal	Observed	g_s/g_i
$g_j(^2P_{3/2}Ga) / g_i(^2P_{1/2}Ga) 2$	2	(1.00172 ± 0.00006)	2 (1.00114 ± 0.00004)
$g_j(^2S_{1/2}Na) / g_i(^2P_{3/2}Ga) 3$	3	(1.00242 ± 0.00006)	2 (1.00121 ± 0.00003)
$g_j(^2S_{1/2}Na) / g_i(^2P_{3/2}In) 3$	3	(1.00243 ± 0.00010)	2 (1.00121 ± 0.00005)

It is to be noted that the ratio g_s/g_i which has been determined has been found from the ratio of the g_j values on the basis of the assumption that the coupling is Russell-Saunders coupling. Hence the deviation of the ratio g_s/g_i from its nominal value of 2 as determined from any pair of atoms or any pair of states does not constitute clear evidence that the spin moment of the electron is other than one Bohr magneton because of the possibility of occurrence of significant perturbations of the states. Theoretical arguments, however, indicate that such perturbations must be small. On experimental grounds the agreement of the ratio obtained in three different ways from different atoms in different spectroscopic states offers overwhelming evidence that the spin moment of the electron does indeed differ from its nom-

inal value by the indicated amount. The discrepancies between the three values of the ratio may, however, arise from perturbations of the indicated energy levels.

A later intercomparison of the g , values of the alkali atoms and a comparison of the g , value of potassium and hydrogen has demonstrated that the g , values of the three alkali atoms of lowest atomic number are indeed equal to the spin g value of the electron to within one part in forty thousand. A further intercomparison by Mann and me¹¹ of the g , values of indium in the $^2P_{1/2}$ and $^2P_{3/2}$ states has given further confirmation to the interpretation of the discrepancy between a measured ratio of g , values and the nominal value.

The experiments that have been described were performed at a field of about 400 gauss. In a wholly independent series of experiments Taub and I¹² determined the ratio of the g , value of indium in the $^2P_{1/2}$ state and that of sodium in the $^2S_{1/2}$ state by observations of lines in the h.f.s. spectrum at fields that ranged from 3,300 to 12,000 gauss. The method was to determine the nuclear g value of the proton in an alkali hydroxide in terms of the g , values of indium and sodium. The result, in so far as it concerns the proton, is of no further interest here in view of the highly refined experiments which have been done in later years that allow the precise and direct determination of the nuclear g value of the proton in terms of both the nuclear and the Bohr magneton. The result is, however, of interest in that it yields again the ratio of the g , values in two different states on the basis of measurements at fields which differed from those in the earlier experiments by an order of magnitude. We found that:

$$g_s / g_l = 2 (1 + 0.00119)$$

We may, therefore, conclude on the basis of all evidence that the electron does indeed possess an « intrinsic » or « anomalous » magnetic moment over and above that deduced from the Dirac theory and whose magnitude is very close to 0.119 percent of the Bohr magneton.

Perhaps it is well, at this point, to make a brief statement of the theoretical status of the spin magnetic moment of the electron. Soon after the publication of our first results which gave substance to the assertion that the electron does have an anomalous moment, Schwinger¹³ gave a result, based on new procedures in quantum electrodynamics, that

$$g_s / g_l = 2 (1 + a / 2\pi) = 2 (1.00116)$$

The result is in excellent agreement with experimental measurements of the same quantity. The effect of the increased electron moment arises essentially as a consequence of the quantization of the electromagnetic field which always has a residual zero-point amplitude. While the existence of this field had previously been recognized, it had not been possible to deal with the interaction prior to the formulation of the contemporary quantum electrodynamics. The importance of the observation of the anomalous magnetic moment of the electron is in part in the demonstration that the procedures of quantum electrodynamics are, in fact, satisfactory in formulating a description of Nature.

It is obvious that a more detailed study of the magnetic moment of the electron than that described thus far was desirable. The objective of a more extended investigation lies in the avoidance of theoretical difficulties in the interpretation, to a high precision, of the electronic g values of complex atoms. In the absence of substantial difficulties of interpretation, the very great precision of which spectroscopy by the method of atomic beams is capable may be used to obtain results of sufficient precision to test the validity of the calculations of quantum electrodynamics when made to a higher order than those originally made by Schwinger.

Barring only a measurement of the spin moment of the free electron itself, the best measurement that one may hope to make is on the electron in the hydrogen atom. In this atom in the ground state, the electron has no orbital angular momentum and hence there is no contribution to the electronic magnetic moment from the orbital motion. The entire electronic magnetic moment arises from the spin moment of the electron. Koenig, Prodell and I¹⁴ have determined the ratio of the electronic g value, g_p , of the hydrogen atom and the nuclear g value of the proton by experimental procedures to be described. To a very high order of accuracy, g is equal to g_s' , the spin g value of the electron bound in the hydrogen atom. The value of g_s' differs from g_s of the free electron through a small relativistic effect of about eighteen parts per million. Corrections due to a mixing of states and relativistic effects are well known and do not limit the accuracy with which the ratio g_s'/g_p may be determined at the present time. Gardner and Purcell¹⁵ have measured the ratio $2g_e/g_p$ of the cyclotron frequency of the electron and the precession frequency of the proton in a magnetic field to an accuracy of about one part in eighty thousand. Our result when combined with that of Gardner and Purcell yields g_s'/g_p .

As a preliminary procedure, Prodell and I¹⁶ determined the hyperfine

structure separation of hydrogen with high precision. A subsequent investigation by both Wittke and Dicke¹⁷ and by us¹⁸ indicated an excessively optimistic estimate of the uncertainty. However, the value of the zero-field h.f.s. splitting of hydrogen that we used was sufficiently good to contribute no error to the value of g_i/g_p , comparable to other uncertainties.

The apparatus designed for the purpose of these experiments had for the magnet which determined the transition frequencies, one with a much better field homogeneity than that which usually characterizes the magnets used in atomic beams experiments. Ordinarily in an atomic beams apparatus, the magnet that determines the splitting of the levels is internal to the vacuum system. This arrangement permits small magnet gaps to be used and hence the production of large fields with electromagnets of moderate dimensions and power consumption. The use of a small gap, however, leads to a considerable hazard of field inhomogeneity. In the present case the magnet was external to the vacuum envelope, the pole faces were of large diameter to reduce edge effects and the magnet could be carefully shimmed after each change of externally imposed experimental parameters and from day to day to give good homogeneity in the volume within which transitions were observed. The deflecting magnets consisted of current-carrying conductors rather than the iron magnets that have become conventional in atomic beams experiments. This choice was made because of the smaller distortion of the transition field by current-carrying conductors than by massive blocks of iron.

The experiment involved the measurement of the frequency of transition between the levels $m = 0$ and $m = -1$ in the state for which $F = I$, alternately with the proton resonance frequency in the same magnetic field. The frequency of the first of these lines is of the order of 3,600 megacycles at a field of 1,500 gauss. The frequency of the proton resonance line is about 6.5 megacycles at the same field and is found by the methods of nuclear resonance in the same region of space as that traversed by the beam. An important component of the equipment is a device that can insert a cylindrical sample of water or mineral oil into a region as closely coincident as inherent limitations permit to that in which the atomic line has been observed. Various small corrections relating to the residual inhomogeneity of the field, bulk diamagnetism of the matter in the cylindrical sample that we employed, to the presence of paramagnetic ions when we observed the resonance in water and the differential internal diamagnetic shielding between oil and water must be applied.

