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General characteristics of radiations emitted by systems moving with super-light velocities with some applications to plasma physics

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The mechanism of radiation of light by a system moving with a super-light velocity is a very simple one and common to the radiation at corresponding conditions of all kinds of waves - electromagnetic as well as sound waves, waves on the surface of water, etc.

Consider a system which in principle is able to emit the radiation in question - e.g. an electrically charged particle in the case of light, a projectile or an airplane in the case of sound, etc. As long as the velocity of this system as a whole is smaller than the velocity of propagation of waves in the surrounding medium, the radiation can be produced only by some oscillatory motion of the system or of some of its parts - e.g. by the oscillation of an electron in an atom or by the revolutions of the propellers of a plane. The frequency of the radiation emitted is evidently determined by the frequency of the oscillations in question. To be more exact, for the radiation to be possible the motion has not necessarily to be a periodic one, but it has to be non-uniform* (i.e. its velocity should not be constant in time).

But when a velocity of the system becomes greater than that of the waves in question, quite a new mechanism of radiation is introduced, by means of which even systems possessing a constant velocity radiate. Let c'(w) denote the velocity of propagation in the surrounding medium of waves, possessing the frequency ω. Then as a rule the radiation of a system moving in the medium with a constant velocity ν, embraces all the frequencies which satisfy the fundamental condition

$$n > c'(w) \qquad (1)$$

* About an exception *to* this rule - the so-called transition radiation - see **V. L. Ginz-**burg and I. Frank[1] (1945).

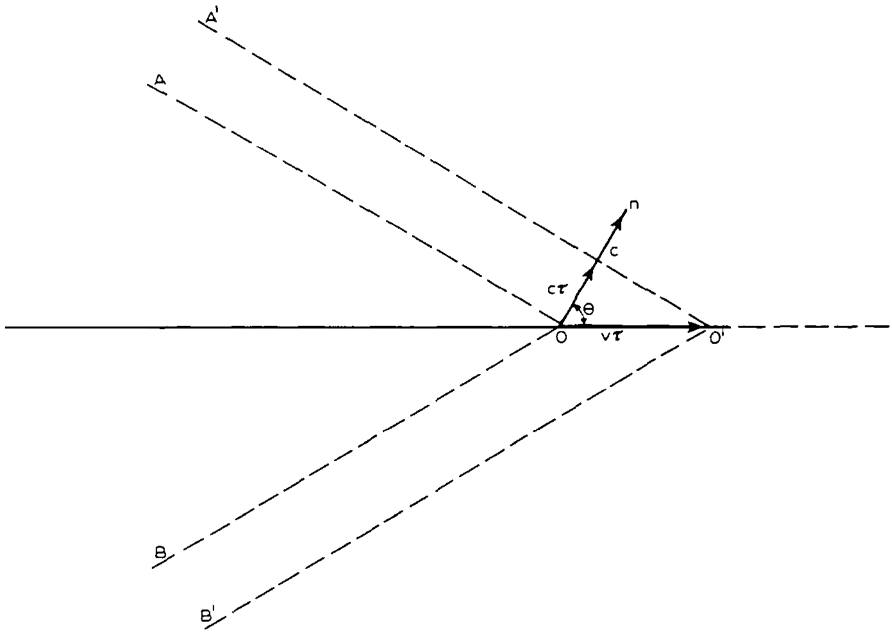

Fig. 1.

This radiation is characteristically a very directional one - waves of a given frequency ω are emitted only under a definite angle Θ to the direction of motion of the system, this angle being determined by the relation

$$\cos\Theta = \frac{c'(\omega)}{v} \tag{2}$$

To prove these fundamental relations one has only to take account of the fact that at all velocities, whether small or large, the field of a uniformly moving system must be stationary with respect to this system. If the system radiates, it means that in its field at least one free wave is present (a free wave of a frequency ω is by definition propagated in the medium with the characteristic phase velocity $c'(\omega)$ to any distance, however far from the source of the wave). Let O and O' (Fig. 1) be the positions of the uniformly moving system at two consecutive moments $t = 0$ and $t = \tau$. The phase of the wave radiated by the system must be stationary with respect to the system. It means, that if AO is that front of the wave* which at the moment

* The fronts of the wave are conical, due to the cylindrical symmetry; AOB is the projection on the plane of drawing of such a cone.

$t = 0$ passes through the system at O, then this front, being propagated in the medium with the velocity $c'(\omega)$, will permanently keep up with the system, and in particular will at the moment $t = \tau$ occupy such a position AO', as to pass through O'. Now the direction $\vec{n}$ of propagation of a free wave is perpendicular to its front, therefore the triangle OCO' is a rectangular one and we easily obtain from it the fundamental relation (2).

Since the value of a cosine cannot exceed unity, Eq. (1) follows directly from (2).

All these general properties of the radiation in question were for a very long time well known in aerodynamics. The air waves emitted at supersonic velocities are called Mach waves. The emission of these waves sets in when the velocity of a projectile or of a plane begins to exceed the velocity of sound in the air. Emitting waves means losing energy and these losses are so large that they constitute the main source of resistance to the flight of a supersonic plane.

That is why in order to cross the sound barrier, i.e. to achieve supersonic velocities in aviation, it was necessary to increase very substantially the power of the engines of a plane.

We perceive the Mach waves radiated by a projectile as its familiar hissing or roaring. That is why, having understood the quite similar mechanism of the Vavilov-Čerenkov radiation of light by fast electrons, we have nicknamed it « the singing electrons ».

I should perhaps explain that we in the USSR use the name « Vavilov-Čerenkov radiation » instead of just « Čerenkov radiation » in order to emphasize the decisive role of the late Prof. S. Vavilov in the discovery of this radiation.

You see that the mechanism of this radiation is extremely simple. The phenomenon could have been easily predicted on the basis of classical electrodynamics many decades before its actual discovery. Why then was this discovery so much delayed? I think that we have here an instructive example of a situation not uncommon in science, the progress of which is often hampered by an uncritical application of inherently sound physical principles to phenomena, lying outside of the range of validity of these principles.

For many decades all young physicists were taught that light (and electromagnetic waves in general) can be produced only by *non-uniform* motions of electric charges. When proving this theorem one has - whether explicitly or implicitly - to make use of the fact, that super-light velocities are forbidden

in a definite direction $\vec{n}$, it necessarily possesses a momentum* ε/c', directed along $\vec{n}$. Therefore the conservation of momentum leads to the vector equation

$$(\varepsilon/c')\vec{n} + \Delta\vec{p} = 0 \tag{4}$$

where $\vec{p}$ is the momentum of the system A. If the increase $\Delta\vec{p}$ of $\vec{p}$ is small in relation to $\vec{p}$, then, according to a general rule,

$$\vec{v}\cdot\Delta\vec{p} = \Delta T \tag{5}$$

Combining these simple and general relations one gets

$$\Delta U = -\varepsilon\left(1 - \frac{v\cos\Theta}{c'}\right) \tag{6}$$

where Θ is the angle between $\vec{v}$ and $\vec{n}$.

If the system A possesses no internal degrees of freedom (e.g. a point charge), then $\Delta U = 0$ and Eq. (6) reduces to the already discussed Eq. (2). Thus we have obtained this fundamental equation once again, but by a new way of reasoning. On the other hand, if the system possesses internal (say, oscillatory) degrees of freedom, and if its velocity is small $(v \ll c')$, then, usual, the internal energy U of the system decreases by an amount equal to the amount ε of the energy radiated.

But at super-light velocities $(v > c')$ the value of the bracket in (6) may become negative, so that radiation of energy by the system may be companied by a *positive* increase $(\Delta U > 0)$ of its internal energy U. example, an atom, being originally in the stable state, radiates light an the same time becomes excited! In such a case the energy both of then tion and of the excitation is evidently borrowed from the kinetic energy i.e. the self-excitation of a system is accompanied by a corresponding slowing down of the motion of this system as a whole.

*For the case of electromagnetic radiation it was shown first by quantum-theoretical reasoning (Ginzburg, 1940) and then by means of classical electrodynamics (Marx and Györgyi[6], 1955) that ε/c' (c' being the phase velocity) is in fact equal to the total

agreement is so good, rather than with explaining existing disagreements. »

Turning again to ordinary plasma I would like to emphasize, that the absorption of plasma waves in the plasma itself is conditioned by a reverse Vavilov-Čerenkov effect.

Ordinarily the necessary condition for a marked absorption of waves is the existence of a resonance between the frequency of the wave and a frequency of the absorbing system, e.g. an atom. Thus a free electron, which in distinction to a bound electron possesses no eigen-frequency, performs in the field of a wave periodic oscillations, alternatively acquiring and again losing kinetic energy and thus producing no substantial absorption.

But there exists also another non-resonant mechanism of absorption. If the velocity v of a free electron is greater than that of the wave (v > c'), then the projection of the velocity of the electron on the direction of propagation of the wave v cos Q may become equal to the velocity of the wave:

$$v \cos Q = c' \qquad (7)$$

In this case the electron so to say rides on the crest of the wave, being exposed to a force, the direction of which does not alter in time, and thus continually absorbs energy from the wave until its velocity increases so much, that it drops out of phase with the wave.

Such is the mechanism of absorption of plasma*; the condition (7), which sorts out those plasma electrons which take part in the process of absorption, is identical with the fundamental condition (2) for radiation**.

The damping coefficient y of plasma waves was first calculated by Landau" in 1946. Changing the notations used by Landau one can present the cxponential term in Landau's formula in the following form

$$\gamma \sim \exp\left(-\frac{mu^2}{2\varkappa T}\right) \qquad (8)$$

* In principle this mechanism of absorption was indicated as long ago as 1949 by Bohm and Gross[11]. The work of these authors is intimately connected with earlier work of A. Vlasov. A detailed and a very lucid mathematical treatment of this subject was presented by R. Z. Sagdeev and V. D. Shafranov at the Geneva Atoms for Peace Conference last September.

** Radiation takes place if there is say one electron of velocity $\vec{v}$ or a cluster of such electrons, the dimensions of the cluster being small in comparison with the length of the wave radiated. If however electrons of a given velocity $\vec{v}$ are distributed continuously in space, then they do not radiate, since their wave-fields are destroyed by mutual interference. But they do absorb.

where $u = \omega_0/k$. In the range of validity of Landau's formula ω_0/k equals the velocity c' of the wave in question.

Therefore according to (8) the damping of a plasma wave is proportional to the density of plasma electrons, possessing according to Maxwell's law a velocity u, equal to the velocity of the wave. This is in exact correspondence to the mechanism of absorption just indicated.

In a recent paper on the mechanism of the sporadic solar radio-emission Ginzburg and Zhelesniakov[10] (1958) applied and extended the theory outlined above to a new and very interesting domain of physics, the foundations of which were laid in Sweden by Professor Alfvén. In particular they have shown that the known instability of a beam of charged particles traversing plasma, is from a quantum theoretical point of view due to the negative absorption of plasma waves by the beam of particles (the *induced* radiation of waves by the beam particles prevailing over the true absorption).

Before finishing I would like to mention one problem, which plays a rather important role in the present fascinating world-wide effort to harness thermonuclear reactions for peaceful uses - the problem how to heat the plasma. First stages of heating can be easily achieved by exciting an electric current in the plasma. However, the cross-section for Coulomb collisions of charged particles decreases inversely to the fourth power of their relative velocities and in a hot and rarefied plasma these collisions become so rare as to become negligible. Evidently heating by electric currents thus becomes impracticable: only a very small part of the energy of the ordered motion of plasma electrons, excited by an external field, is under these conditions converted into Joule heat.

Many different methods to achieve further heating of the plasma are now being discussed, e.g. the so-called magnetic pumping. I wish to make some remarks on only two such methods, intimately connected with our subject.

First, the heating by a beam of fast charged particles, injected into plasma from outside, is in principle feasible even if the plasma is hot and rarefied. Although in such a plasma energy losses of fast particles due to close collisions become negligible, coherent energy losses, described earlier, are independent of the collision cross-section and become all-important.

It is necessary to stress in this connection two points. First, the heating can in principle be achieved by a beam of fast charged particles travelling not in the plasma itself, but outside it and parallel to its surface. In fact, as we have seen, coherent energy losses are due to the emission of plasma waves