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# Relativistic quantum field theory

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The relativistic quantum theory of fields was born some thirty-five years ago through the paternal efforts of Dirac, Heisenberg, Pauli and others. It was a somewhat retarded youngster, however, and first reached adolescence seventeen years later, an event which we are gathered here to celebrate. But it is the subsequent development and more mature phase of the subject that I wish to discuss briefly today.

I shall begin by describing to you the logical foundations of relativistic quantum field theory. No dry recital of lifeless « axioms » is intended but, rather, an outline of its organic growth and development as the synthesis of quantum mechanics with relativity. Indeed, relativistic quantum mechanics - the union of the complementarity principle of Bohr with the relativity principle of Einstein - *is* quantum field theory. I beg your indulgence for the mode of expression I must often use. Mathematics is the natural language of theoretical physics. It is the irreplaceable instrument for the penetration of realms of physical phenomena far beyond the ordinary experience upon which conventional language is based.

Improvements in the formal presentation of quantum mechanical principles, utilizing the concept of action, have been interesting by-products of work in quantum field theory. Both my efforts in this direction[1] and those of Feynman[2] (which began earlier) were based on a study of Dirac concerning the correspondence between the quantum transformation function and the classical action. We followed quite different paths, however, and two distinct formulations of quantum mechanics emerged which can be distinguished as differential and integral viewpoints.

In order to suggest the conceptual advantages of these formulations, I shall indicate how the differential version transcends the correspondence principle and incorporates, on the same footing, two different kinds of quantum dynamical variable. It is just these two types that are demanded empirically by the two known varieties of particle statistics. The familiar properties of the variables $q_k, p_k$, $k = 1 \cdots n$, of the conventional quantum system enable one to *derive* the form of the quantum action principle. It is a differential statement about time transformation functions,

$$\delta\langle t_1|t_2\rangle = (i/\hbar)\langle t_1|\delta[\int_{t_2}^{t_1} dtL]|t_2\rangle \tag{1}$$

which is valid for a certain class of kinematical and dynamical variations. The quantum Lagrangian operator of this system can be given the very symmetrical form

$$L = \sum_{k=1}^{n} \frac{1}{4}\left(p_k \frac{dq_k}{dt} - q_k \frac{dp_k}{dt} + \frac{dq_k}{dt} p_k - \frac{dp_k}{dt} q_k\right) - H(q,p,t) \tag{2}$$

The symmetry is emphasized by collecting all the variables into the $2n$-component Hermitian vector $z(t)$ and writing

$$L = \frac{1}{4}\left(za\frac{dz}{dt} - \frac{dz}{dt}az\right) - H(z,t) \tag{3}$$

where $a$ is a real antisymmetrical matrix, which only connects the complementary pairs of variables.

The transformation function depends explicitly upon the choice of terminal states and implicitly upon the dynamical nature of the system. If the latter is held fixed, any alteration of the transformation function must refer to changes in the states, as given by

$$\delta\langle t_1| = (i/\hbar)\langle t_1|G_1 \qquad \delta|t_2\rangle = -(i/\hbar)\, G_2|t_2\rangle \tag{4}$$

where $G_1$ and $G_2$ are infinitesimal Hermitian operators constructed from dynamical variables of the system at the specified times. For a given dynamical system, then,

$$\delta[\int_{t_2}^{t_1} dtL] = G_1 - G_2 \tag{5}$$

which is the quantum principle of stationary action, or Hamilton's principle, since there is no reference on the right hand side to variations at intermediate times. The stationary action principle implies equations of motion for the dynamical variables and supplies explicit expressions for the infinitesimal operators $G_{1,2}$. The interpretation of these operators as generators of transformations on states, and on the dynamical variables, implies commutation relations. In this way, all quantum-dynamical aspects of the system are derived from a single dynamical principle. The specific form of the commutation relations obtained from the symmetrical treatment of the usual quantum system is given by the matrix statement

$$[z(t), z(t)] = i\hbar a^{-1} \tag{6}$$

finitesimal neighborhood of a point. The characteristic time derivative or kinematical part of L appears analogously in $\mathscr{L}$ in terms of the variables associated with the specified spatial point. The relativistic structure of the action principle is completed by demanding that it present the same form, independently of the particular partitioning of space-time into space and time. This is facilitated by the appearance of the action operator, the time integral of the Lagrangian, as the space-time integral of the Lagrange function. Accordingly, we require, as a sufficient condition, that the latter be a scalar function of its field variables, which implies that the known form of the time derivative term is supplemented by similar space derivative contributions. This is conveyed by

$$\mathscr{L} = \frac{1}{4}\left(\chi A^{\mu}\partial_{\mu}\chi - \partial_{\mu}\chi A^{\mu}\chi\right) - \mathscr{H}(\chi) \tag{11}$$

where the $A^{\mu}$ are a set of four finite skew-Hermitian matrices. A specific physical field is associated with submatrices of the $A^{\mu}$, which are real and anti - symmetrical for a field j that obeys Bose-Einstein statistics, or imaginary and symmetrical for a field $\psi$ obeying Fermi-Dirac statistics. Finally, the boundaries of the four-dimensional integration region, formed by three-dimensional space at the terminal times, are described by the invariant concept of the space-like surface $\sigma$, a three-dimensional manifold such that every pair of points is in space-like relation. The ensuing invariant form of the action principle of relativistic quantum field theory is (we now use atomic units, in which $\hbar = c = 1$)

$$\delta\langle\sigma_1|\sigma_2\rangle = i\langle\sigma_1|\delta\left[\int_{\sigma_2}^{\sigma_1}(\mathrm{d}x)\,\mathscr{L}\right]|\sigma_2\rangle \tag{12}$$

Relativity is a statement of equivalence within a class of descriptions associated with similar but different measurement apparatus. Space-time coordinates are an abstraction of the role that the measurement apparatus plays in defining a space-time frame of reference. The empirical fact, that all connected space-time locations and orientations of the measurement apparatus supply equivalent descriptions, is interpreted by the mathematical requirement ofinvariance under the group of proper orthochronous inhomogenous Lorentz transformations, applied to the continuous numerical coordinates. There is another numerical element in the quantum- mechanical description that has a measure of arbitrariness and expresses an aspect of relativity. I am referring to the quantum-mechanical use of complex numbers and of the mathematical equivalence of the two square roots of - 1, ± i. What general property of any measurement apparatus is subject to our control, in principle, but offers only the choice of two alternatives? The answer is clear - a macroscopic material system can be constructed of matter, or of antimatter! But let us not conclude too hastily that a matter apparatus and an antimatter apparatus are completely equivalent. It is characteristic of quantum mechanics that the dividing line between apparatus and system under investigation can be drawn somewhat arbitrarily, as long as the measurement apparatus always possesses the classical aspects required for the unambiguous recording of an observation. To preserve this feature, the interchange of matter and antimatter must be made on the whole assemblage of macroscopic apparatus and microscopic system. Since the observational label of this duality is the algebraic sign of electric charge, the microscopic interchange must reverse the vector of electric current $j^{\mu}$, while maintaining the tensor $T^{\mu\nu}$ that gives the flux of energy and momentum. But this is just the effect of the coordinate transformation that reflects all four coordinates.

It is indeed true that the action principle does not retain its general form under either of the two transformations, the replacement of *i* with - *i*, and the reflection of all coordinates, but does preserve it under their combined influence. In more detail, the effect of complex conjugation is equivalent to the reversal of operator multiplication, which distinguishes fields with the two types of statistics. The reflection of all coordinates, a proper transformation, can be generated by rotations in the attached Euclidean space obtained by introducing the imaginary time coordinate $x_4 = i\,x^0$. This transformation alters reality properties, distinguishing fields with integral and half-integral spin. The combination of the two transformations replaces the original Lagrange function

$$\mathscr{L}(\varphi_{\text{int}}, \varphi_{1/2\,\text{int}}, \psi_{\text{int}}, \psi_{1/2\,\text{int}}) \tag{13}$$

with

$$\mathscr{L}(\varphi_{\text{int}}, i\varphi_{1/2\,\text{int}}, i\psi_{\text{int}}, \psi_{1/2\,\text{int}}) \tag{14}$$

If only fields of the types $\varphi_{\text{int}}, \Psi_{\frac{1}{2}\,\text{int}}$ are considered, which is the empirical connection between spin and statistics, the action principle is unaltered in form. This invariance property of the action principles expresses the relativity of matter and antimatter. That is the content of the so-called TCP theorem. The anomalous response of the field types $\varphi_{\frac{1}{2}\,\text{int}}, \Psi_{\text{int}}$ is also the basis for the theoretical rejection of these possibilities as contrary to general physical requirements of positiveness, namely, the positiveness of probability, and the positiveness of energy.