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# Energy production in stars

*Nobel Lecture, December 11, 1967*

## *History*

From time immemorial people must have been curious to know what keeps the sun shining. The first scientific attempt at an explanation was by Helmholtz about one hundred years ago, and was based on the force most familiar to physicists at the time, gravitation. When a gram of matter falls to the sun's surface it gets a potential energy

$$E_{pot} = -GM/R = -1.91 \cdot 10^{15} \text{ erg/g} \quad (1)$$

where $M = 1.99 \cdot 10^{33}$ g is the sun's mass, $R = 6.96 \cdot 10^{10}$ cm its radius, and $G = 6.67 \cdot 10^{-8}$ the gravitational constant. A similar energy was set free when the sun was assembled from interstellar gas or dust in the dim past; actually somewhat more, because most of the sun's material is located closer to its center, and therefore has a numerically larger potential energy. One-half of the energy set free is transformed into kinetic energy according to the well-known virial theorem of mechanics. This will permit us later to estimate the temperature in the sun. The other half of the potential energy is radiated away. We known that at present the sun radiates

$$\varepsilon = 1.96 \text{ erg/g sec} \quad (2)$$

Therefore, if gravitation supplies the energy, there is enough energy available to supply the radiation for about $10^{15}$ sec which is about 30 million years.

This was long enough for nineteenth century physicists, and certainly a great deal longer than man's recorded history. It was not long enough for the biologists of the time. Darwin's theory of evolution had just become popular, and biologists argued with Helmholtz that evolution would require a longer time than 30 million years, and that therefore his energy source for the sun was insufficient. They were right.

At the end of the 19th century, radioactivity was discovered by Becquerel and the two Curie's who received one of the first Nobel prizes for this discovery. Radioactivity permitted a determination of the age of the earth, and more recently, of meteorites which indicate the time at which matter in the solar system solidified. On the basis of such measurements the age of the sun is estimated to be 5 milliards of years, within about 10%. So gravitation is not sufficient to supply its energy over the ages.

Eddington, in the 1920's, investigated very thoroughly the interior constitution of the sun and other stars, and was much concerned about the sources of stellar energy. His favorite hypothesis was the complete annihilation of matter, changing nuclei and electrons into radiation. The energy which was to be set free by such a process, if it could occur, is given by the Einstein relation between mass and energy and is

$$c^2 = 9 \cdot 10^{20} \text{ erg/g} \tag{3}$$

This would be enough to supply the sun's radiation for 1500 milliards of years. However nobody has ever observed the complete annihilation of matter. From experiments on earth we know that protons and electrons do not annihilate each other in $10^{30}$ years. It is hard to believe that the situation would be different at a temperature of some 10 million degrees such as prevails in the stars, and Eddington appreciated this difficulty quite well.

From the early 1930's it was generally assumed that the stellar energy is produced by nuclear reactions. Already in 1929, Atkinson and Houtermans[1] concluded that at the high temperatures in the interior of a star, the nuclei in the star could penetrate into other nuclei and cause nuclear reactions, releasing energy. In 1933, particle accelerators began to operate in which such nuclear reactions were actually observed. They were found to obey very closely the theory of Gamow, Condon and Gurney, on the penetration of charged particles through potential barriers. In early 1938, Gamow and Teller[2] revised the theory of Atkinson and Houtermans on the rate of « thermonuclear » reactions, *i.* e. nuclear reactions occurring at high temperature. At the same time, von Weizsäcker[3] speculated on the reactions which actually might take place in the stars.

In April 1938, Gamow assembled a small conference of physicists and astrophysicists in Washington, D. C. This conference was sponsored by the Department of Terrestrial Magnetism of the Carnegie Institution. At this Conference, the astrophysicists told us physicists what they knew about the internal constitution of the stars. This was quite a lot, and all their results had been derived without knowledge of the specific source of energy. The only assumption they made was that most of the energy was produced « near » the center of the star.

*Properties of Stars*

The most easily observable properties of a star are its total luminosity and its surface temperature. In relatively few cases of nearby stars, the mass of the star can also be determined.

Fig.1 shows the customary Hertzsprung-Russell diagram. The luminosity, expressed in terms of that of the sun, is plotted against the surface temperature, both on a logarithmic scale. Conspicuous is the main sequence, going from upper left to lower right, i.e. from hot and luminous stars to cool and faint ones. Most stars lie on this sequence. In the upper right are the Red Giants, cool but brilliant stars. In the lower left are the White Dwarfs, hot but faint. We shall be mainly concerned with the main sequence. After being assembled, by gravitation, stars spend the most part of their life on the main sequence, then develop into red giants, and in the end, probably into white dwarfs.

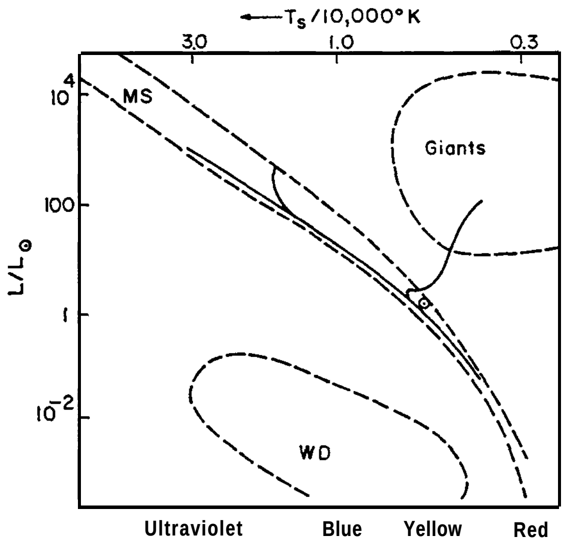


Fig.1. Hertzsprung-Russell diagram. From E.E.Salpeter, in *Apollo and the Universe*, Science Foundation for Physics, University of Sydney, Australia, 1967.

The figure shows that typical surface temperatures are of the order of $10^4$ °K. Fig. 2 gives the relation between mass and luminosity in the main sequence. At the upper end, beyond about 15 sun masses, the mass determinations are uncertain. It is clear, however, that luminosity increases rapidly with mass.

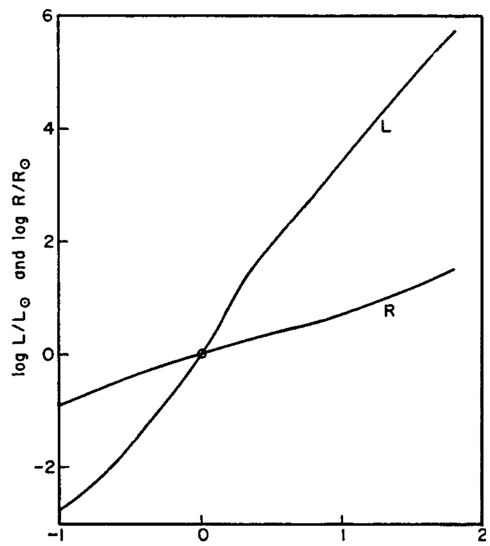


Fig. 2. Luminosity and radius of stars vs. mass. Abscissa is $\log M/M_{\odot}$. Data from C. W. Allen, *Astrophysical* Quantities, Athlone Press, 1963, p. 203. The curve for $\log L/L_{\odot}$ holds for all stars, that for $R/R_{\odot}$ only for the stars in the main sequence. The symbol $\odot$ refers to the sun.

For a factor of 10 in mass, the luminosity increases by a factor of about 3000, hence the energy production per gram is about 300 times larger.

To obtain information on the interior constitution of the stars, astrophysicists integrate two fundamental equations. Pioneers in this work have been Eddington, Chandrasekhar and Strömgren. The first equation is that of hydrostatic equilibrium

$$\frac{dP}{dr} = -GM(r)\frac{\varrho(r)}{r^2} \tag{4}$$

in which $P$ is the pressure at distance $r$ from the center, $\varrho$ is the density and $M(r)$ is the total mass inside $r$. The second equation is that of radiation transport

$$\frac{1}{\varkappa\varrho}\frac{d}{dr}\left(\tfrac{1}{3}acT^4\right) = -\frac{L(r)}{4\pi r^2} \tag{5}$$

Here $\varkappa$ is the opacity of the stellar material for black-body radiation of the local temperature $T$, $a$ is the Stefan-Boltzmann constant, and $L(r)$ is the flux of radiation at $r$. The value of $L$ at the surface $R$ of the star is the luminosity.

In the stars we shall discuss, the gas obeys the equation of state

$$P = RT\varrho/\mu \tag{6}$$

where $R$ is the gas constant, while $\mu$ is the mean molecular weight of the stellar material. If X, Y and Z are respectively concentrations by mass of hydrogen, helium and all heavier elements, and if all gases are fully ionized, then

$$\mu^{-1} = 2X + \tfrac{3}{4}Y + \tfrac{1}{2}Z \tag{7}$$

In all stars except the very oldest ones, it is believed that Z is between 0.02 and 0.04; in the sun at present, X is about 0.65, hence Y= 0.33 and $\mu$=0.65. In many stars the chemical composition, especially X and Y, vary with position *r*. The opacity is a complicated function of *Z* and *T*, but in many cases it behaves like

$$\varkappa = C\varrho\, T^{-3\ 4} \tag{8}$$

where C is a constant.

The integration of (4) and (5) in general requires computers. However an estimate of the central temperature may be made from the virial theorem which we mentioned in the beginning. According to this, the average thermal energy per unit mass of the star is one-half of the average potential energy. This leads to the estimate of the thermal energy per particle at the center of the star,

$$k\,T_c = \alpha\,\mu\,G\,H\,M/R \tag{9}$$

in which *H* is the mass of the hydrogen atom, and $\alpha$ is a constant whose magnitude depends on the specific model of the star but is usually about 1 for main sequence stars. Using this value, and (1), we find for the central temperature of the sun

$$T_{6c} = 14 \tag{10}$$

where $T_6$ denotes the temperature in millions of degrees, here and in the following. A more careful integration of the equations of equilibrium by Demarque and Percy[4] gives

$$T_{c6} = 15.7;\ \varrho_c = 158\ \text{g/cm}^3 \tag{11}$$

Originally Eddington had assumed that the stars contain mainly heavy elements, from carbon on up. In this case $\mu$= 2 and the central temperature is increased by a factor of 3, to about 40 million degrees; this led to contradictions with the equation of radiation flow, ( 5), if the theoretical value of the opacity was used. Strömgren pointed out that these contradictions can be resolved by assuming the star to consist mainly of hydrogen, which is also in agreement with stellar spectra. In modern calculations the three quantities *X, Y, Z,* indicating the chemical composition of the star, are taken to be parameters to be fixed so as to fit all equations of stellar equilibrium.

### *Thermonuclear Reactions*

All nuclei in a normal star are positively charged. In order for them to react they must penetrate each others Coulomb potential barrier. The wave mechanical theory of this shows that in the absence of resonances, the cross section has the form

$$a(E) = \frac{S(E)}{E} \exp\left(-\sqrt{\frac{E_G}{E}}\right) \qquad (12)$$

where $E$ is the energy of the relative motion of the two colliding particles, $S(E)$ is a coefficient characteristic of the nuclear reaction involved and

$$E_G = 2M(\pi Z_0 Z_1 e^2/\hbar)^2 = (2\pi Z_0 Z_1)^2 E_{\mathrm{Bohr}} \qquad (13)$$

Here $M$ is the reduced mass of the two particles, $Z_0$ and $Z_1$, their charges, and $E_{\mathrm{Bohr}}$ is the Bohr energy for mass $M$ and charge 1. (13 ) can be evaluated to give

$$E_G = 0.979\ W \text{ MeV} \qquad (14)$$

with

$$W = A Z_0^2 Z_1^2 \qquad (14a)$$

$$A = A_0 A_1/(A_0 + A_1) \qquad (14b)$$

in which $A_0$, $A_1$ are the atomic weights of the two colliding particles. For most nuclear reactions $S(E)$ is between 10 MeV-barns and 1 keV-barn.

The gas at a given $r$ in the star has a given temperature so that the particles have a Boltzmann energy distribution. The rate of nuclear reactions is then proportional to

$$(8/\pi M)^{\frac{1}{2}} (kT)^{-3/2} \int \sigma(E)\, E \exp(-E/kT)\, \mathrm{d}E \qquad (15)$$

It is most convenient[5] to write for the rate of disappearance of one of the reactants

$$\mathrm{d}X_0/\mathrm{d}t = -[01]\, X_0 X_1 \qquad (16)$$

where $X_0$ and $X_1$ are the concentrations of the reactants by mass, and

$$[01] = 7.8\cdot10^{11}\,(Z_0\,Z_1/A)^{1/3}\,S_{\text{eff}}\,\varrho\,T_6^{-2/3}\,e^{-\tau} \tag{17}$$

$$\tau = 42.487\,(W/T_6)^{1/3} \tag{17a}$$

Since the reaction cross section (12) increases rapidly with energy, the main contribution to the reaction comes from particles which have an energy many times the average thermal energy. Indeed the most important energy is

$$E_0 = (\tau/3)\,k\,T \tag{18}$$

For $T = 13$ which is an average for the interior of the sun, we have

$$\begin{aligned} \tau/3 = 4.7 &\text{ for the reaction H + H} \\ 19 &\text{ for the reaction C + H} \\ 25 &\text{ for the reaction N + H} \end{aligned} \tag{19}$$

It is also easy to see from (17) that the temperature dependence of the reaction rate is

$$\frac{d\,\ln[01]}{d\ln T} = \frac{\tau-2}{3} \tag{20}$$

*Nuclear Reactions in Main Sequence Stars*

Evidently, at a given temperature and under otherwise equal conditions, the reactions which can occur most easily are those which have the smallest possible value of $W$ (14a). This means that at least one of the interacting nuclei should be a proton, $A_0 = Z_0 = 1$. Thus we may examine the reactions involving protons.

The simplest of all possible reactions is

$$H + H = D + \varepsilon^+ + \nu \tag{21}$$

($\varepsilon^+$ = positron, $\nu$ = neutrino).

This was first suggested by von Weizsäcker[3], and calculated by Critchfield and Bethe[6]. The reaction is of course exceedingly slow because it involves the beta disintegration. Indeed the characteristic factor $S$ is

$$S(E) = 3.36\cdot10^{-25}\text{ MeV-barns} \tag{22}$$

This has been derived on purely theoretical grounds, using the known coupling constant of beta disintegration; the value is believed to be accurate to 20% or better. There is no chance of observing such a slow reaction on earth, but in the stars we have almost unlimited time, and a large supply of protons of high energy. As we shall see presently, the rate of energy production by this simple reaction fits the observed energy production in the sun very well.

The deuterons formed in (21) will quickly react further, and the end product is $^4$He. We shall discuss the reactions in more detail later on.

The proton-proton reaction (21), although it predicts the correct energy production in the sun, has a rather weak dependence on temperature. According to (19), (20), it behaves about as $T^4$. Since central temperatures change only little from the sun to more massive stars, the energy production by this reaction does likewise. However as we have seen in Fig. 2, the observed energy production increases dramatically with increasing mass. Therefore there must exist nuclear reactions which are more strongly dependent on temperature; these must involve heavier nuclei.

Stimulated by the Washington Conference of April 1938, and following the argumentjust mentioned, I examined[7] the reactions between protons and other nuclei, going up in the periodic system. Reactions between H and $^4$He lead nowhere, there being no stable nucleus of mass 5. Reactions of H with Li, Be and B, as well as with deuterons, are all very fast at the central temperature of the sun, but just this speed of the reaction rules them out: the partner of *H* is very quickly used up in the process. In fact, and just because of this reason, all the elements mentioned, from deuterium to boron, are extremely rare on earth and in the stars, and can therefore not be important sources of energy.

The next element, carbon, behaves quite differently. In the first place, it is an abundant element, probably making up about 1% by mass of any newly formed star. Secondly, in a gas of stellar temperature, it undergoes a cycle of reactions, as follows

$$^{12}C + H = {}^{13}N + \gamma \quad (23a)$$

$$^{13}N = {}^{13}C + \varepsilon^+ + \nu \quad (23b)$$

$$^{13}C + H = {}^{14}N + \gamma \quad (23c)$$

$$^{14}N + H = {}^{15}O + \gamma \quad (23d)$$

$$^{15}O = {}^{15}N + \varepsilon^+ + \nu \quad (23e)$$

$$^{15}N + H = {}^{12}C + {}^4He \quad (23f)$$

Reactions a, c, and d are radiative captures; the proton is captured by the nucleus and the energy emitted in the form of gamma rays; these are then quickly converted into thermal energy of the gas. For reactions of this type, *S (E)* is of the order of 1 keV- barn. Reactions b and e are simply spontaneous beta decays, with lifetimes of 10 and 2 min respectively, negligible in comparison with stellar times. Reaction f is the most common type of nuclear reaction, with 2 nuclei resulting from the collision; *S (E)* for such reactions is commonly of the order of MeV- barns.

Reaction f is in a way the most interesting because it closes the cycle: we reproduce the $^{12}C$ which we started from. In other words, carbon is only used as a catalyst; the result of the reaction is a combination of 4 protons and 2 electrons[8] to form one $^{4}He$ nucleus. In this process two neutrinos are emitted, taking away about 2 MeV energy together. The rest of the energy, about 25 MeV per cycle, is released usefully to keep the sun warm.

Making reasonable assumptions of the reaction strength *S (E),* on the basis of general nuclear physics, I found in 1938 that the carbon-nitrogen cycle gives about the correct energy production in the sun. Since it involves nuclei of relatively high charge, it has a strong temperature dependence, as given in (19). The reaction with $^{14}N$ is the slowest of the cycle and therefore determines the rate of energy production; it goes about as $T^{24}$ near solar temperature. This is amply sufficient to explain the high rate of energy production in massive stars[9].

## *Experimental Results*

To put the theory on a firm basis, it is important to determine the strength factor *S (E)* for each reaction by experiment. This has been done under the leadership of W.A. Fowler[10] of the California Institute of Technology in a monumental series of papers extending over a quarter of a century. Not only have all the reactions in (23) been observed, but in all cases *S (E)* has been accurately determined.

The main difficulty in this work is due to the resonances which commonly occur in nuclear reactions. Fig. 3 shows the cross section of the first reactions (23a), as a function of energy. The measured cross sections extend over a factor of $10^{7}$ in magnitude; the smallest ones are $10^{-11}$ barns= $10^{-35}$ $cm^{2}$ and therefore clearly very difficult to observe. The curve shows a resonance at 460 keV. The solid curve is determined from nuclear reaction theory, on the basis of the existence of that resonance. The fit of the observed points to the calculated curve is impressive. Similar results have been obtained on the other three proton-capture reactions in (23).

On the basis of Fig. 3 we can confidently extrapolate the measurements to lower energy. As we mentioned in (18) the most important energy contribut -

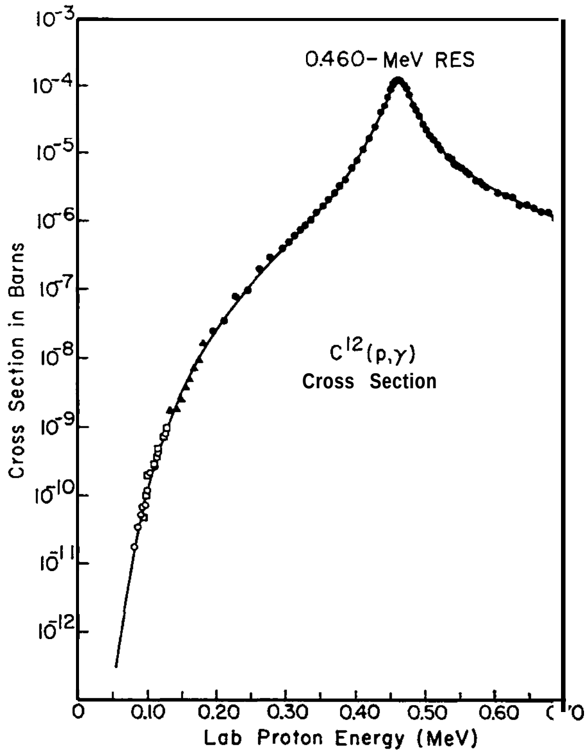


Fig. 3. Cross section for the reaction $^{12}C$ + H, as a function of the proton energy. From Fowler, Caughlan and Zimmerman[5].

ing to the reaction rate is about 20 $kT$. For $T_6 = 13$, we have $kT = 1.1$ keV; so we are most interested in the cross section around 20 keV. This is much too low an energy to observe the cross section in the laboratory; even at 100 keV, the cross section is barely observable. So quite a long extrapolation is required. This can be done with confidence provided there are no resonances close to $E = 0$. Therefore a great deal of experimental work has gone into the search for such resonances.

The resonances exist of course in the compound nucleus, *i. e.* the nucleus obtained by adding the two initial reactants. To find resonances near the threshold of the reactions (23), it is necessary to produce the same compound nucleus from other initial nuclei, e.g., in the reaction between $^{14}N$ and H, the compound nucleus $^{15}O$ is formed. To investigate its levels Hensley[11] at CalTech studied the reaction

$$^{16}O + {}^{3}He = {}^{15}O + {}^{4}He \qquad (24)$$

He found indeed a resonance 20 keV below the threshold for $^{14}N$ + H which in principle might enhance the process (23d). However the state in $^{15}O$ was found to have a spin $J$= 7/2. Therefore, even though $^{14}H$ has $J$= 1 and the proton has a spin of 1/2, we need at least an orbital momentum $\lambda$= 2, to reach this resonant state in $^{15}O$. The cross section for such a high orbital momentum is reduced by at least a factor $10^4$, compared to $\lambda$=0, so that the near-resonance does not in fact enhance the cross section $^{14}N$+H appreciably. This cross section can then be calculated by theoretical extrapolation from the measured range of proton energies, and the same is true for the other reactions in the cycle (23).

On this basis, Fowler and others have calculated the rate of reactions in the CN cycle. A convenient tabulation has been given by Reeves[12]; his results are plotted in Fig.4. This figure gives the energy production per gram per second as a function of temperature. We have assumed X= 0.5, $Z$= 0.02.

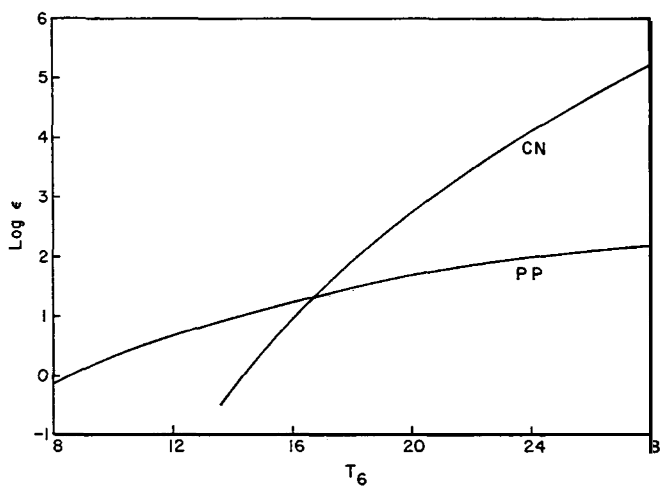


Fig. 4. The energy production, in erg/g sec as a function of the temperature in millions of degrees. For the proton-proton reaction (PP) and the carbon-nitrogen cycle (CN). Concentrations assumed X= Y= 0.5, $Z$= 0.02. Calculated from Tables 8 and 9 of Reeves[12].

The figure shows that at low temperature the H + H reaction dominates, at high temperatures the C + N cycle; the crossing point is at $T_6$= 13; here the energy production is 7 erg/g sec. The average over the entire sun is obviously smaller, and the result is compatible with an average production of 2 erg/g sec.

The energy production in the main sequence can thus be considered as well understood.

An additional point should be mentioned. Especially at higher temperature, when the CN cycle prevails, there is also a substantial probability for the reaction chain

$$^{16}O + H = {}^{17}F + \gamma \tag{25a}$$

$$^{17}F = {}^{17}O + \varepsilon^+ + \nu \tag{25b}$$

$$^{17}O + H = {}^{14}N + {}^{4}He \tag{25c}$$

This chain is not cyclic but feeds into the CN cycle. It is customary to speak of the whole set of reactions as the CNO bi-cycle. The effect of reactions (25) is that $^{16}O$ initially present will also contribute to the reactants available, and thus increase the reaction rate of the CN cycle somewhat. This has been taken into account in Fig. 5.

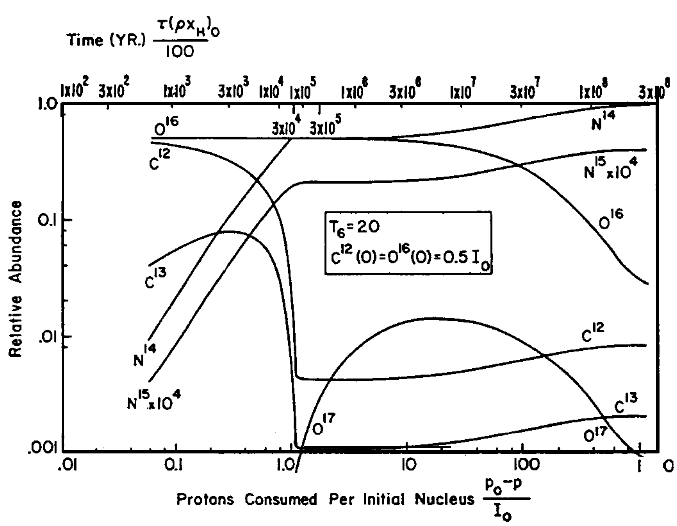


Fig. 5. Variation with time of the abundances of various elements involved in the CNO cycle. It is assumed that initially $^{12}C$ and $^{16}O$ have the same abundance while that of $^{14}N$ is small. From G.R. Caughlan, *Astrophys.].* (1967) .

If equilibrium is established in the CNO bi-cycle, eventually most of the nuclei involved will end up as $^{14}N$ because this nucleus has by far the longest lifetime against nuclear reactions. There is no observable evidence for this; in fact wherever the abundance can be observed, C and O tend to be at least as abundant as N. However this is probably due to the fact that the interior of a star stays well separated from its surface; there is very little mixing. Astrophysicists have investigated the circumstances when mixing is to be expected, and have found that surface abundances are quite compatible with these expectations. In the interstellar material which is used to form stars, we have reason to believe that C and O are abundant and N is rare. This will be discussed later.

### *The Completion of the Proton-Proton Chain*

The initial reaction (21) is followed almost immediately by

$$^{2}H + H = {}^{3}He + \gamma \tag{26}$$

The fate of $^{3}$He depends on the temperature. Below about $T_6 = 15$, the $^{3}$He builds up sufficiently so that such nuclei react with each other according to

$$2\ {}^{3}He = {}^{4}He + 2\ H \tag{27}$$

This reaction has an unusually high $S(E) = 5$ MeV- barns[5]. At higher temperature, the reaction

$$^{4}He + {}^{3}He = {}^{7}Be + \gamma \tag{28}$$

competes favorably with (27). The $^{7}$Be thus formed may again react in one of two ways

$$^{7}Be + e^{-} = {}^{7}Li + \nu \tag{29a}$$

$$^{7}Be + H = {}^{8}B + \gamma \tag{29b}$$

$$^{8}B = 2\ {}^{4}He + \varepsilon^{+} + \nu \tag{29c}$$

At about $T_C = 20$, reaction (29b) begins to dominate over (29a). (29b) is followed by (29c) which emits neutrinos of very high energy. Davies[13], at Brookhaven, is attempting to observe these neutrinos.

### *Evolution of a Star*

A main sequence star uses up its hydrogen preferentially near its center where nuclear reactions proceed most rapidly. After a while, the center has lost almost all its hydrogen. For stars of about twice the luminosity of the sun, this happens in less than $10^{10}$ years which is approximately the age of the universe, and also the age of stars in the globular clusters. We shall now discuss what happens to a star after it has used up the hydrogen at the center. Of course, in the outside regions hydrogen is still abundant.

This evolution of a star was first calculated by Schwarzschild[14] who has been followed by many others; we shall use recent calculations by Iben[15]. When hydrogen gets depleted, not enough energy is produced near the center to sustain the pressure of the outside layers of the star. Hence gravitation will cause the center to collapse. Thereby, higher temperatures and densities are achieved. The temperature also increases farther out where there is still hydrogen left, and this region now begins to burn. After a relatively short time, a shell of *H,* away from the center, produces most of the energy; this shell gradually moves outward and gets progressively thinner as time goes on.

At the same time, the region of the star outside the burning shell expands. This result follows clearly from all the many numerical computations on this subject. The physical reason is not clear. One hypothesis is that it is due to the discontinuity in mean molecular weight: Inside the shell, there is mostly helium, of m= 4/3, outside we have mostly hydrogen, and m= 0.65. Another suggestion is that the flow of radiation is made difficult by the small radius

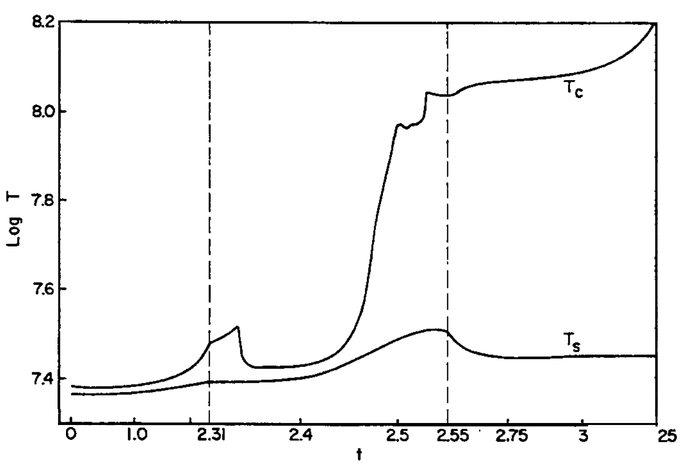


Fig. *6.* Evolution of a star of 3 solar masses, according to I. Iben, *Astrophys.J.,* 142 *(1965)* 1447. Abscissa is time in units of $10^8$ years (note the breaks in scale at *t= 2.31* and 2.55). I. Temperature (on logarithmic scale) : $T_c$= temperature at center of star, $T_s$= same at mid-point of source of energy generation, which, after *t = 2.48* is a thin shell. $T_c$, increases enormously, $T_s$ stays almost constant.

of the energy source, and that this has to be compensated by lower density ust outside the source.

By this expansion the star develops into a red giant. Indeed, in globular clusters (which, as I mentioned, are made up of very old stars), all the more luminous stars are red giants. In the outer portion of these stars, radiative transport is no longer sufficient to carry the energy flow; therefore convection of material sets in in these outer regions. This convection can occupy as much as the outer 80% of the mass of the star; it leads to intimate mixing of the material in the convection zone.

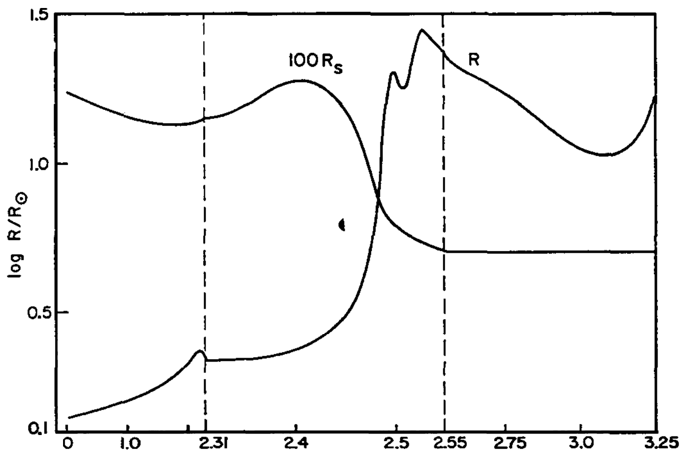


Fig. 7. Evolution of a star, (see caption to Fig. 6). II Radius, in units of that of the sun, on logarithmic scale. $R$ = total radius, 100 $R_S$ = 100 times the radius of mid-point of energy source. $R$ increases tremendously, while $R$, shrinks somewhat.

Iben[15] has discussed a nice observational confirmation of this convectional mixing. The star Capella is a double star, each component having a mass of about 3 solar masses, and each being a red giant. The somewhat lighter star, « Capella F » (its spectral type is F) shows noticeable amounts of Li in its spectrum, while the somewhat heavier Capella G shows at least 100 times less Li. It should be expected that G, being heavier, is farther advanced in its evolution. Iben now gives arguments that the deep-reaching convection and mixing which we just discussed, will occurjust between the evolution phases F and G. By convection, material from the interior of the star will be carried to the surface; this material has been very hot and has therefore burned up its Li. Before deep convection sets in (in star F) the surface Li never sees high temperature and thus is preserved.

Following the calculations of Iben we have plotted in Figs. 6-9 the development of various important quantities in the history of a star of mass= 3 solar masses. The time is in units of $10^8$ years. Since the developments go at very variable speed, the time scale has been broken twice, at $t$= 2.31 and $t$= 2.55. In between is the period during which the shell source develops.

During this period the central temperature rises spectacularly (Fig. 6) from about $T_6$= 25 to $T_6$= 100. At the same time the radius increases from about 2 to 30 solar radii; subsequently, it decreases again to about 15 (Fig. 7). The central density, starting at about 40, increases in the same period to about $5 \cdot 10^4$(Fig.8). The luminosity (Fig. 9) does not change spectacularly, staying always between 100 and 300 times that of the sun.

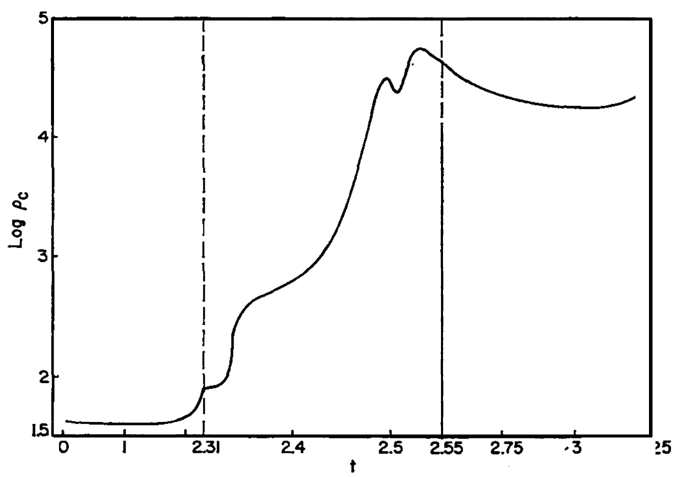


Fig. 8. Evolution of a star (see caption to Fig. 6).111. Density, on logarithmic scale, at the center of the star. This quantity increases about 1000-fold.

While the inside and the outside of the star undergo such spectacular changes, the shell in which the hydrogen is actually burning, does not change very much. Fig. 9 shows $m$, the fraction of the mass of the star enclosed by the burning shell. Even at the end of the calculation, $t$= 3.25, this is only $m$= 0.2. This means that only 20% of the hydrogen in the star has burned after all this development. Fig. 6, curve $T_s$, shows the temperature in the burning shell which stays near 25 million degrees all the time. Fig. 7, curve $R_s$, shows the radius of the shell, in units of the solar radius; during the critical time when the shell is formed this radius drops from about 0.15 to 0.07. This is of course the mechanism by which the shell is kept at the temperature which originally prevailed at the center.