Quantum Coherence between States with Even and Odd Numbers of Electrons

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A system with variable number of electrons is described in which the states representing coherent superpositions of states with even and odd numbers of electrons may occur. An experiment is suggested which generalizes the experiment of Nakamura *et al.* and may provide direct evidence of such coherence and, thereby, justify the reality of a superspace.

Quantum coherence between states with even and odd numbers of electrons is of special fundamental interest.

In 1952, Wick, Wightman, and Wigner [1] claimed that the coherent linear superpositions of states with even and odd numbers of fermions are incompatible with the Lorentz invariance and introduced the superselection rule, according to which such linear superpositions are physically impossible. In actuality (as was pointed out in [2, 3]), the superselection rule is the alternative to the existence, along with x, y, z, and t, of additional spinor coordinates, which, in fact, are introduced in quantum field theory to account for supersymmetry.

In this work, it is proved theoretically that the superselection rule is, generally, not self-consistent. Namely, a simple realistic system with variable number of electrons is considered, which is governed by the Hamiltonian whose eigenvectors are all coherent superpositions of the states with even and odd numbers of electrons. The idea of the proof is as follows.

The number of electrons is a conserved quantity which is analogous, in this respect, to the momentum and the angular momentum. Physical systems with Hamiltonians whose eigenvectors are all coherent superpositions of the states with different momenta are well known. A particle in an external potential field depending on the particle coordinate is the simplest system of this type. In fact, this particle is part of an isolated system consisting of the particle and a certain massive body, the interaction with which can be described as an external field acting on the particle. As known, this is justified only if certain requirements are fulfilled. For example, the states of a massive body must adiabatically adjust to the changes in the particle coordinate in order to prevent excitation of the body degrees of freedom.

Below, the system with variable number of electrons is considered which form, together with the environment, an isolated common system with a fixed number of electrons. The interaction of the system with the environment (an analogue of a massive body in the above-mentioned example) can be described as an external field acting on the system. In the case considered, the interaction does not commute with the operator of number of electrons in the system (by analogy with the fact that the potential energy of interaction between a particle and a massive body does not commute with the particle momentum operator). Moreover, this field has the spinor character; *i.e.*, it changes sign under the $\mathbf{O}(2\pi)$ and $\mathbf{R}^2$ transformations, where $\mathbf{O}(2\pi)$ is the rotation of the coordinate system through an angle of 2π about any axis and $\mathbf{R}^2$ is the double time reversal. All eigenfunctions of the Hamiltonian of the system are coherent superpositions of the states with even and odd numbers of electrons.

Below, an experiment will also be proposed which generalizes the Nakamura *et al.* experiment [4] on the observation of quantum coherence between the states with different (but even in both cases) numbers of electrons. The implementation of this experiment will directly demonstrate the coherence between the states with even and odd numbers of electrons.

1. Let us consider a simple example where the interaction of the two parts of a total closed system can be described in terms of spinor external fields.

Let there be two quantum dots and one electron which can occur with close energies in either of the dots and has a certain spin projection, specified once and for all. The quantum dots have gates, whose electric potentials can be varied to move the electron energy levels. In the second quantization representation, a complete set of system states $|n, N\rangle$ consists of two states:

$$|0,1\rangle\,, \quad |1,0\rangle\,. \tag{1}$$

Here, n and N are the numbers of electrons in the first dot (which will be referred to as system) and the second dot (referred to as environment), respectively. The characteristic feature of states (1) is that the set of environment quantum numbers N is uniquely determined by the quantum numbers of the system: $N = 1 - n$. It is this property that allows the interaction of the system with environment to be treated as an external field acting on the system. If we are interested only in the state of the system itself, we can consider any state $|n, 1-n\rangle$ of the common system as the state

$$|n\rangle = |n, 1-n\rangle$$

of our system (the first dot) interacting with the environment. Moreover our system is characterized by a certain Hamiltonian.

With allowance made for electron tunneling between the quantum dots, the Hamiltonian of the total system is

$$H = H_0 + H_t, \tag{2}$$

where

$$H_0 = e(n) + E(N), \tag{3}$$

In Eq.(3), $e(n)$ and $E(N)$ are the energies of quantum dots in the gate potentials without tunneling, and

$$H_t = VaA^+ - V^*a^+A, \tag{4}$$

where V is the tunneling amplitude; a, a^+ and A, A^+ are the operators of electron annihilation and creation in the system and the environment, respectively. In the usual representation (see [6], Sec.65), the action of the operators A and A^+ on vectors (1) is given by the formulas (if the result is nonzero)

$$A\,|0,1\rangle = |0,0\rangle\,, \quad A^+\,|1,0\rangle = -\,|1,1\rangle\,. \tag{5}$$

The substitution of Eq.(5) into Hamiltonian (4) shows that the tunneling Hamiltonian in the case of interest is equivalent to the following interaction Hamiltonian involving only the system operators:

$$H_\eta = \eta a + \eta^* a^+. \tag{6}$$

Here, the operators a and a^+ act on the vectors $|n\rangle$ by the usual rules of an isolated system, and η is the external field equal to $-V$ in the representation used. The total Hamiltonian of the system is then

$$H = e(n) + E(1-n) + H_\eta, \tag{7}$$

so that the total interaction energy is the sum of the second and third terms in Eq.(7). Under the $\mathbf{O}(2\pi)$ and $\mathbf{R}^2$ transformations, the field η, as well as the operators a and other spinor quantities, change sign so that, for a given field value, Hamiltonian (6) is not invariant about these transformations. Due to the presence of terms linear in electron operators, all eigenstates of the Hamiltonian are coherent superpositions of the states with even and odd numbers of electrons.

2. The two-level system described by Eq.(7) obeys the time-dependent Shrödinger equations

$$i\dot{a} = \tau b, \quad i\dot{b} = \epsilon b + \tau^* a, \tag{8}$$

where $\tau = -V$ and $\epsilon = e(1) - e(0) + E(0) - E(1)$. The general state of this two-level system is $a\,|0\rangle + b\,|1\rangle$.

The experiment of Nakamura *et al.* [4] is as follows. Before the initial time ($t = 0$), a two-level system was in the ground state with the gate potential such that $\epsilon \gg \tau$. Accordingly, $a = 1$ and $b = 0$. At $t = 0$, the gate potential rapidly changes to a value for which $\epsilon = 0$. Then, the potential remains constant for a time Δt, after which it rapidly regains its initial value. On the time interval between $t = 0$ and $t = \Delta t$, the system obeys Eqs. (8) with $\epsilon = 0$ and initial conditions $a(t = 0) = 1$ and $b(t = 0) = 0$. Then $a(t) = \cos|\tau|t$, and $\bar{b}(t) = \sin|\tau|t$, where $\bar{b} \equiv (i\tau/|\tau|)b$ so that $|\bar{b}| = |b|$. At $t = \Delta t + 0$, one measures the excited-state population

$$|b(\Delta t)|^2 = \frac{1}{2}(1 - \cos 2|\tau|\Delta t) \tag{9}$$

as a function of pulse duration Δt. This can be done (as in the experiment of Nakamura *et al.* [4]) using a probe electrode connected to the box through a tunneling contact or (as in the experiment of Aassime *et al.* [6]) using a probe electrometer based on a single-electron transistor. The observed oscillations indicate that the system coherently oscillates between the states with electron numbers 0 and 2 on the time interval $(0, \Delta t)$. If such an experiment had been performed with our two-level system, it would have been proved that the corresponding system coherently oscillate between the states with electron numbers 0 and 1. It should be emphasized that this interpretation of the oscillations is essentially based on the description of the system by the Hamiltonian (7), which account for the interaction with environment by introducing the field η. However, if one interprets oscillations (9) as a phenomenon occurring in the total closed system, then they are evidence only of the oscillatory electron transitions between different parts of the system. According to this interpretation, the oscillation frequency is proportional to $|\tau|$, *i.e.*, to the tunneling amplitudes. One may note, in this connection, that the experiment of Nakamura *et al.* can be refined by passing from the single-pulse to two-pulse technique (See [7]).

As above, let the two-level system be at $t < 0$ in the ground state $a = 1$ and $b = 0$ ($\epsilon \gg |\tau|$). The amplitude of the first gate-potential rectangular pulse is the same as above (*i.e.*, corresponds to $\epsilon = 0$), but its duration is fixed at $t_1 = \pi/4|\tau|$. Immediately after the pulse at $t = t_1 + 0$, the system is in the state with $a = \bar{b} = 1/\sqrt{2}$. In the interval between $t = t_1$ and $t = t_1 + \Delta t$, the potential is equal to its initial value corresponding to $\epsilon \gg |\tau|$. Under these conditions, the tunneling interaction of the system with environment can be ignored and it behaves as a closed system in its pure state. In this case, $a(t) = 1/\sqrt{2}$ and $\bar{b}(t) = (1/\sqrt{2})\exp(i\phi(t))$, with the relative phase of the ground and excited states linearly depending on time: $\phi(t) = -\epsilon(t - t_1)$.

However, it would be incorrect to conceive that the environment is also in the pure state and the state vector of the total system is the product of the state vectors of its parts. In this case, the total system occurs in the so-called entangled state (See [8]).

The second gate-potential pulse with parameters of the first pulse is switched on at time $t_1 + \Delta t$. Using Eqs.(8), one can see that, after completion of the second pulse at time $2t_1 + \Delta t$ ($\epsilon \gg |\tau|$ because $\Delta t \ll t_1$), the population of the excited state is

$$|b|^2 = \frac{1}{2}(1 + \cos \epsilon \Delta t). \tag{10}$$

The observation of oscillations (10) as a function of time delay Δt between the pulses would demonstrate that the relative phase of the states with different numbers of electrons in a closed system has a definite value $\phi(t)$ linearly depending on time. For our two-level system, this would be direct proof of the quantum coherence between the states with even and odd numbers of electrons.

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