KUNGL. VETENSKAPSAKADEMIEN
THE ROYAL SWEDISH ACADEMY OF SCIENCES

Information Department, P.O. Box 50005, SE-104 05 Stockholm, Sweden
Phone: +46 8 673 95 00, Fax: +46 8 15 56 70, E-mail: info@kva.se, Website: www.kva.se

Superfluids and superconductors: quantum mechanics on a macroscopic scale

Superfluidity or superconductivity – which is the preferred term if the fluid is made up of charged particles like electrons – is a fascinating phenomenon that allows us to observe a variety of quantum mechanical effects on the macroscopic scale. Besides being of tremendous interest in themselves and vehicles for developing key concepts and methods in theoretical physics, superfluids have found important applications in modern society. For instance, superconducting magnets are able to create strong enough magnetic fields for the magnetic resonance imaging technique (MRI) to be used for diagnostic purposes in medicine, for illuminating the structure of complicated molecules by nuclear magnetic resonance (NMR), and for confining plasmas in the context of fusion-reactor research. Superconducting magnets are also used for bending the paths of charged particles moving at speeds close to the speed of light into closed orbits in particle accelerators like the Large Hadron Collider (LHC) under construction at CERN.

Discovery of three model superfluids

Two experimental discoveries of superfluids were made early on. The first was made in 1911 by Heike Kamerlingh Onnes (Nobel Prize in 1913), who discovered that the electrical resistance of mercury completely disappeared at liquid helium temperatures. He coined the name "superconductivity" for this phenomenon. The second discovery – that of superfluid ^{4}He – was made in 1938 by Pyotr Kapitsa and independently by J.F. Allen and A.D. Misener (Kapitsa received the 1978 Nobel Prize for his inventions and discoveries in low temperature physics). It is believed that the superfluid transition in ^{4}He is a manifestation of Bose-Einstein condensation, *i.e.* the tendency of particles – like ^{4}He – that obey Bose-Einstein statistics to condense into the lowest-energy single–particle state at low temperatures (the strong interaction between the helium atoms blurs the picture somewhat). Electrons, however, obey Fermi-Dirac statistics and are prevented by the Pauli principle from having more than one particle in each state. This is why it took almost fifty years to discover the mechanism responsible for superconductivity. The key was provided by John Bardeen, Leon Cooper and Robert Schrieffer, whose 1957 "BCS theory" showed that pairs of electrons with opposite momentum and spin projection form "Cooper pairs". For this work they received the 1972 Nobel Prize in Physics. In their theory the Cooper pairs are structureless objects, i.e. the two partners form a spin-singlet in a relative *s*-wave orbital state, and can to a good approximation be thought of as composite bosons that undergo Bose-Einstein condensation into a condensate characterised by macroscopic quantum coherence.

Since both the Cooper pairs of the original BCS theory and the helium atoms are spherically symmetric objects, they form *isotropic* superfluids on condensation. The situation is more complex – and therefore more interesting – in a third model superfluid discovered by David Lee, Douglas Osheroff and Robert Richardson in 1972. Their discovery of superfluidity in ^{3}He was rewarded by a Nobel Prize in 1996. While ^{4}He is a boson, ^{3}He with three rather than four nucleons is a fermion, and superfluid ^{3}He is formed by condensation of Cooper pairs of ^{3}He atoms (or more precisely of "quasiparticles" of atoms each with a surrounding cloud of other atoms) that have internal degrees of freedom. This is because the two partners form a spin-triplet in a relative orbital *p*-state. Both the net spin of the pair and their relative orbital momentum are therefore different from zero and the superfluid is intrinsically *anistropic*; roughly speaking, each pair carries two vectors that can point in various directions as will be discussed below.

Broken symmetry and the order parameter

Even before the discovery of superfluid ^{3}He, theoreticians had been interested in anistropic superfluids. In order to appreciate their significance it is useful to recall the importance of the concepts of *order parameter* and *spontaneously broken symmetry* in the theory of superfluidity. The concept of an order parameter was introduced by Lev Landau in connection with his 1937 theory of second order phase transitions. The order parameter is a quantity that is zero in the disordered phase above a critical temperature T_c, but has a finite value in the ordered state below T_c. In the theory of ferromagnetism, *e.g.,* spontaneous magnetisation, which is zero in the magnetically disordered paramagnetic state and nonzero in the spin-ordered ferromagnetic state, is chosen to be the order parameter of the ferromagnetic state. Clearly, the existence of a preferred direction of the spins implies that the symmetry of the ferromagnet under spin rotation is reduced ("broken") when compared to the paramagnet. This is the phenomenon called spontaneously (*i.e.* not caused by any external field) broken symmetry. It describes the property of a macroscopic system in a state that does not have the full symmetry of the underlying microscopic dynamics.

In the theory of superfluidity the order parameter measures the existence of Bose condensed particles (Cooper pairs) and is given by the probability amplitude of such particles. The interparticle forces between electrons, between ^{4}He and between ^{3}He atoms, are rotationally invariant in spin and orbital space and, of course, conserve particle number. The latter symmetry gives rise to a somewhat abstract symmetry called "gauge symmetry", which is broken in any superfluid. In the theory of isotropic superfluids like a BCS superconductor or superfluid ^{4}He, the order parameter is a complex number Ψ with two components, an amplitude $|\Psi|$ and a phase ("gauge") ϕ. Above T_c the system is invariant under an arbitrary change of the phase $\phi \rightarrow \phi'$, *i.e.* under a gauge transformation. Below T_c a particular value of ϕ is spontaneously preferred.

Multiple simultaneously broken continuous symmetries

In anistropic superfluids, additional symmetries can be spontaneously broken, corresponding to an order parameter with more components. In ^{3}He – the best studied example with a parameter having no fewer than 18 components – the pairs are in a spin-triplet state, meaning that rotational symmetry in spin space is broken, just as in a magnet. At the same time, the anisotropy of the Cooper-pair wave function in orbital space calls for a spontaneous breakdown of orbital rotation symmetry, as in liquid crystals. Including the gauge symmetry, three symmetries are therefore broken in superfluid ^{3}He. The 1972 theoretical discovery that several *simultaneously* broken symmetries can appear in condensed matter was made by **Anthony Leggett**, and represented a breakthrough in the theory of anisotropic superfluids. This leads to superfluid phases whose properties cannot be understood by simply adding the properties of systems in which each symmetry is broken individually. Such phases may have long range order in combined, rather than individual degrees of freedom, as illustrated in Fig. 1. An example is the so-called A phase of superfluid ^{3}He. Leggett showed, for example, that what he called spontaneously broken spin-orbit symmetry leads to unusual properties that enabled him to identify this phase with a particular microscopic state, the ABM state (see below).

The microscopic 1957 BCS theory of superconductivity represents a major breakthrough in the understanding of *isotropic* charged superfluids (superconductors). The original theory does not, however, address the properties of *anisotropic* superfluids (like superfluid ^{3}He, high temperature superconductors and heavy fermion superfluids), which were treated much later with decisive contributions from Leggett and others. Neither is the BCS theory able to describe inhomogeneous superfluids with an order parameter that varies in space as may happen, for example, in the presence of a magnetic field. A particularly important example of such a phenomenon is the type of superconductor used in the powerful superconducting magnets mentioned earlier. Here superconductivity and magnetism coexist. The theoretical description of this very important class of superconductors relies on a phenomenological theory developed in the 1950s by **Alexei Abrikosov**, building on previous work by **Vitaly Ginzburg** and Lev Landau (Landau, who received the Nobel Prize in physics in 1962, died in 1968).

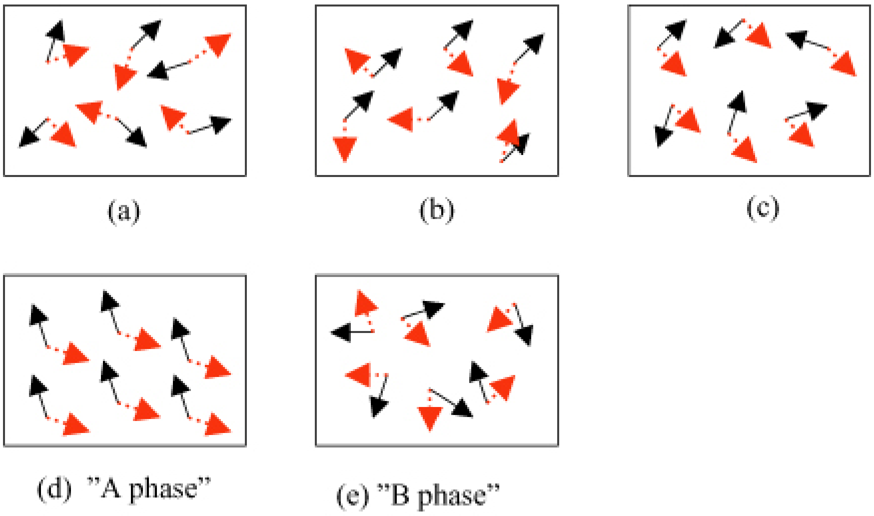

Figure 1. The possible states in a two-dimensional model liquid of particles with two internal degrees of freedom: spin (full-line arrow) and orbital angular momentum (broken-line arrow). (a) Disordered state: isotropic with respect to the orientation of both degrees of freedom. The system is invariant under separate rotations in spin and orbital space and has no long range order (paramagnetic liquid). (b)–(e) States with different types of long range order corresponding to all possible broken symmetries. (b) Broken rotational symmetry in spin space (ferromagnetic liquid). (c) Broken rotational symmetry in orbital space ("liquid crystal"). (d) Rotational symmetries in both spin and orbital space *separately* broken (as in the A phase of superfluid ^{3}He). (e) Only the symmetry related to the *relative* orientation of the spin and orbital degrees of freedom is broken (as in the B phase of superfluid ^{3}He). Leggett introduced the term spontaneosuly broken spin-orbit symmetry for the broken symmetry leading to the ordered states in (d) and (e).

Superconductivity and magnetism

Superconductivity is characterised by electron pairs (or holes) that have condensed into a ground state, where they all move coherently. This means not only that the resistance disappears but also that a magnetic field is expelled from the superconductor (the charged superfluid). This is known as the Meissner effect. Many superconductors show a complete Meissner effect, which means that a transition from the superconducting to the normal state occurs discontinuously at a certain critical external magnetic field H_c. Other superconductors, in particular alloys, only show a partial Meissner effect or none at all. Work done in Kharkov by A. Shubnikov and by others elsewhere showed that the magnetisation may change continuously as the external field is increased, starting at a *lower critical field*, H_{c1}, while the superconductor continues to show no resistance up to a much higher *upper critical field*, H_{c2}. This effect is illustrated in Fig. 2. Between the lower and the upper critical fields the superconducting state coexists with a magnetic field.

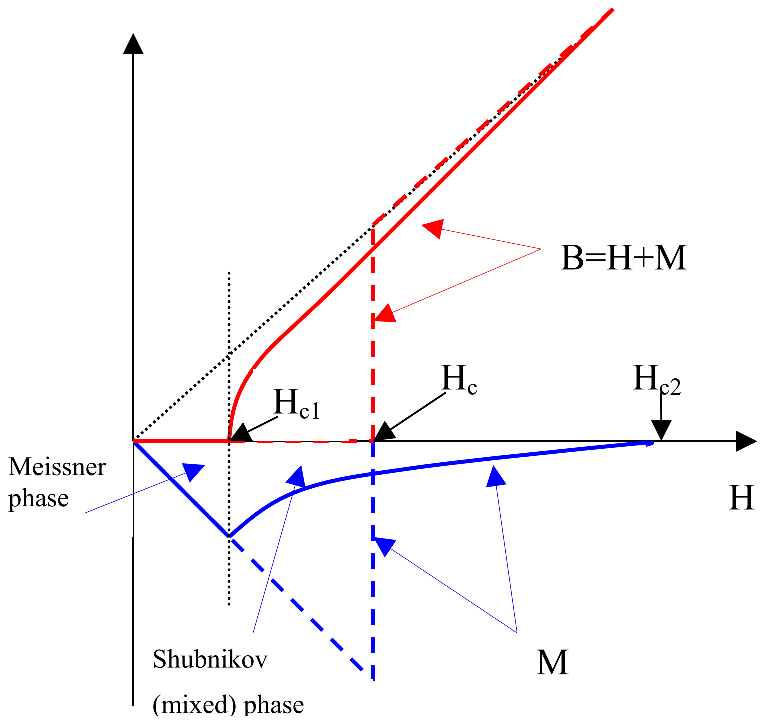

Figure 2. (Colour) Magnetisation M (blue) and induced field B (red) as a function of external magnetic field H for superconductors with complete (dashed lines) and partial (full lines) Meissner effect (see text).

The theoretical framework for understanding the behavior of superconductors in the presence of such strong magnetic fields was developed in the 1950s by a group of Soviet physicists. In a groundbreaking paper, published in 1957, Abrikosov discovered the vortices in the order parameter of a superconductor and described their crucial role for the coexistence of a magnetic field and superconductivity in superconductors "of the second group", or in "type-II superconductors" as we would say today. In the same paper, Abrikosov provided an amazingly detailed prediction – later to be borne out by experiments – of the way in which a stronger magnetic field suppresses superconductivity: vortices, which form a lattice, come closer to each other, and at some field the vortex cores overlap, suppressing the order parameter everywhere in the superconducting material – hence driving it into the normal state.

Abrikosov's results came from an insightful analysis of the Ginzburg-Landau equations, a phenomenological description of superconductivity published in 1950 by Vitaly Ginzburg and Lev Landau. One of the motivations behind their work was the need to develop a theory that would make it possible to describe correctly the destruction of superconductivity by a magnetic field or an electric current. The Ginzburg-Landau equations have proved to be of great importance in physics, not only for describing superconductivity in the presence of a magnetic field. In their 1950 paper Ginzburg and Landau were the first to realize that superconductors can be divided into two classes with regard to their behaviour in a magnetic field. They introduced a quantity κ, now known as the *Ginzburg-Landau parameter*, which enabled them to make a distinction between the two classes. Superconductors with $\kappa < 1/\sqrt{2}$ do not allow the coexistence of a magnetic field and superconductivity in the same volume. Superconducting materials with $\kappa > 1/\sqrt{2}$ do allow for such a coexistence. In modern language $\kappa = \lambda/\xi$ is the ratio of the magnetic field penetration length λ and the coherence length ξ.

The superconductors known at the time had $\kappa << 1$, *e.g.*, $\kappa \approx 0.16$ for mercury. That is why Ginzburg and Landau did not seriously pursue this parameter region beyond showing that if a material with $\kappa > 1/\sqrt{2}$ is placed in a magnetic field somewhat larger than the thermodynamic critical value, the normal phase is unstable with respect to formation of a superconducting state. However, they introduced the crucial notions of a superconducting order parameter, of negative surface energy of the boundary separating the superconducting from the normal phase in type-II superconductors, and (in modern terminology) of the upper critical magnetic field, where superconductivity vanishes in type-II materials. Even so, it was left to Abrikosov to describe in 1957 the result of this instability and to formulate the complete phenomenological theory of type-II superconductors. At the same time it is clear that the Ginzburg-Landau equation and the partial understanding achieved by Ginzburg and Landau was a necessary basis for his work.

Below we describe the main contributions of Abrikosov, Ginzburg and Leggett, the 2003 Nobel Physics Laureates, in some more detail. We will do this in the chronological order the contributions were made. (Readers who want to skip the next three, somewhat technical sections, can go directly to the last section on the importance of the contributions.)

Ginzburg-Landau theory

When Ginzburg and Landau formulated their phenomenological theory of superconductivity in 1950, almost 50 years had passed since Kamerlingh Onnes discovered the superfluid electron liquid in mercury. This was well before the BCS theory but a certain level of understanding had been reached using phenomenological methods. Early on, Gorter and Casimir introduced the two-fluid model (a similar model was developed for superfluid helium). They divided the conduction electrons into two groups, a superconducting condensate and normal electrons excited from the condensate. Later, in 1935, the brothers Fritz and Heinz London presented a phenomenological theory that could explain why a magnetic field does not penetrate further into a metal than the *London penetration depth,* λ, a concept we have already alluded to. However, the London theory could not describe correctly the destruction of superconductivity by a magnetic field or current. Nor did it allow a determination of the surface tension between the superconducting and normal phases in the same material (Landau had in 1937 assumed the surface tension to be positive in his theory of the so called intermediate state). Neither could the London theory explain why the critical magnetic fields needed to destroy superconductivty in thin films are different from the critical fields for bulk superconductors of the same material. These deficiencies provided the motivation for Ginzburg and Landau. Their phenomenological Ginzburg-Landau theory of superconductivity was indeed able to solve these problems.

The Ginzburg-Landau (GL) theory is based on Landau's theory of second order phase transitions from 1937. This was a natural starting point, since in the absence of a magnetic field the transition into the superconducting state at a critical temperature T_c is a second-order phase transition. Landau's theory describes the transition from a disordered to an ordered state in terms of an "order parameter", which is zero in the disordered phase and nonzero in the ordered phase. In the theory of ferromagnetism, for example, the order parameter is the spontaneous magnetisation. In order to describe the transition to a superconducting state, GL took the order parameter to be a certain complex function $\Psi(r)$, which they interpreted as the "effective" wave function of the "superconducting electrons", whose density n_s is given by $|\Psi|^2$; today we would say that $\Psi(r)$ is the macroscopic wave function of the superconducting condensate.

In accordance with Landau's general theory of second-order phase transitions, the free energy of the superconductor depends only on $|\Psi|^2$ and may be expanded in a power series close to T_c. Assuming first that $\Psi(\mathbf{r})$ does not vary in space, the free energy density becomes

$$f_s = f_n + \alpha|\Psi|^2 + (\beta/2)|\Psi|^4 + ...$$

where the subscripts n and s refer to the contributions from the normal and the superconducting state respectively. A stable superconducting state is obtained if β is a positive constant and $\alpha = \alpha_0 (T - T_c)$.

Since the purpose of Ginzburg and Landau was to describe the superconductor in the presence of a magnetic field, H, when the order parameter may vary in space, gradient terms had to be added to the expansion. The lowest order gradient term looks like a kinetic energy term in quantum mechanics, which is why GL wrote it – adding a term for the magnetic field energy – as

$$\frac{1}{2m^*}\left|\left(-i\hbar\nabla - e^* A\right)\Psi(r)\right|^2 + \frac{1}{2\mu_0}(\nabla \times A)^2$$

Here the magnetic field $H = (\nabla \times A)/\mu_0$ is described by its vector potential, $\boldsymbol{A}(\boldsymbol{r})$, which enters the kinetic energy term as required by gauge-invariance. The total free energy F_s is obtained by integrating the free energy density f_s over volume.

By minimising the free energy F_s with respect to Ψ, Ψ^* and A, the GL equations are obtained. They are

$$\frac{1}{2m^*}\left(-i\hbar\nabla - \frac{e^*}{c}A\right)^2 \Psi + \alpha\Psi + \beta|\Psi|^2\Psi = 0,$$

$$j = \nabla \times H = \frac{e^*\hbar}{2m^*c}\left(\Psi^*\nabla\Psi - \Psi\nabla\Psi^*\right) + \frac{e^{*2}}{m^*c^2}|\Psi|^2 A$$

plus a boundary condition.

The second equation has the same form as the usual expression for the current density in quantum mechanics, while the first – except for a term nonlinear in Ψ, which acts like a repulsive potential – resembles the Schrödinger equation for a particle of mass *m**, charge *e** with energy eigenvalue $-\alpha$. In their paper Ginzburg and Landau wrote that "*e** is the charge, which there is no reason to consider as different from the electronic charge". As soon as they learned about the BCS theory and Cooper pairs, however, they realized that $e^*=2e$ and $m^*=2m$.

The GL equations are capable of describing many phenomena. An analysis shows, for example, that a magnetic field penetrating into a superconductor decays with its distance from the border to a normal phase region over a characteristic length $\lambda(T)$, where $\lambda^2(T)=\beta m^*/|\alpha|e^{*2}$. This is the London penetration length. Furthermore, it is found that a disturbance $\delta\Psi$ from an equilibrium value of the order parameter, decays over a characteristic length ξ, where $\xi^2(T) = \hbar^2/4m^*|\alpha|$. Therefore, the penetration length λ and the coherence length ξ are two characteristic lengths in the GL theory. (Although the physics was clear to them, Ginzburg and Landau used neither this notation nor this terminology; the concept of a *coherence length* was only introduced three years later by B. Pippard). The two lengths have the same temperature dependence close to T_c, where $\lambda, \xi \propto 1/\sqrt{T_c - T}$.

In 1950 Ginzburg and Landau made a number of predictions for the critical magnetic field and critical current density for thin superconducting films and the surface energy between superconducting and normal phases of the same material. These predictions could soon be tested experimentally with positive results.

At this point a short digression about the surface energy between superconducting and normal phases of the same material is called for. It follows from the GL equations that this quantity depends on the two characteristic lengths λ and ξ in a way that can be understood from Fig. 3. The penetration of the magnetic field, a distance of the order λ, into the superconductor corresponds to a gain in energy, which is proportional to λ and due to the decreased distortion of the field. On the other hand, the fact that the superconducting state vanishes over a distance of the order ξ close to the border decreases the gain in condensation energy, and hence gives an energy increase proportional to ξ. The net surface energy is the sum of the two contributions and can be expressed as $(\xi/\sqrt{2} - \lambda)\mu_0 H_c^2/2$. In terms of the

Ginzburg-Landau parameter $\kappa = \lambda / \xi$ we see that the surface energy is positive if $\kappa < 1/\sqrt{2}$ and negative if $\kappa > 1/\sqrt{2}$. Ginzburg and Landau were mainly interested in clean metals for which κ is much smaller than unity. Nevertheless, they did note this fact and pointed out that there is a "peculiar" instability of the normal phase of the metal if $\kappa > 1/\sqrt{2}$, which is associated with this negative surface energy.

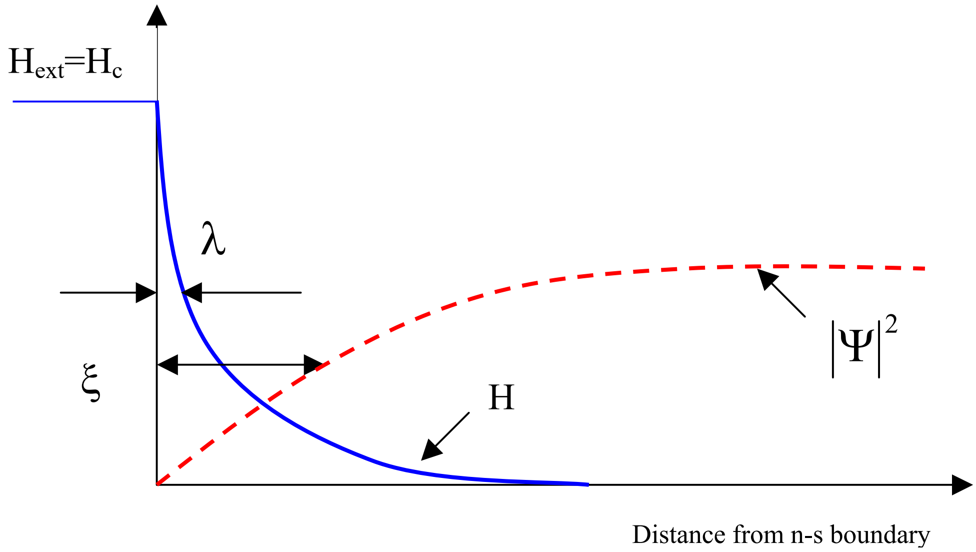

Figure 3. Sketch of the border region between a normal and a superconducting phase, illustrating the concepts of penetration length λ and coherence length ξ. If the magnetic field is H_c in the normal phase, it decays to zero in the superconducting phase over a length λ. At the same time the superconducting order increases from zero at the interface to its full value inside the superconducting phase over a distance ξ.

Theory of type-II superconductors

One of the physicists who soon began to test the predictions of the GL theory was the young N.V. Zavaritzkii. Working at the Kapitsa Institute for Physical Problems in Moscow, he was able to verify the theoretical predictions about the dependence on film thickness and temperature of the critical magnetic field of superconducting films. However, when he tried to make better samples by a new technique (vapour deposition on glass substrates at low temperatures) he discovered that the critical fields no longer agreed with the GL theory. He brought this to the attention of his room mate at the university, Alexei Abrikosov. Abrikosov looked for a solution to this mystery within the GL theory and started to think about the true nature of the superconducting state for $\kappa > 1/\sqrt{2}$. In contrast to the superconductors that were the focus of Ginzburg's and Landau's interest in 1950, the new materials had values in this parameter regime. In 1952 Abrikosov was able to calculate the critical magnetic fields for this parameter regime and found agreement with Zavaritzkii's measurements.

Abrikosov continued to think about strongly “type-II superconductors” with large values of κ. It was clear that superconductivity could not exist in magnetic fields of a certain strength. But Abrikosov was able to show that when the field is diminished again, small superconducting regions start to nucleate at a magnetic field $H_{c2} = H_c\kappa\sqrt{2}$, which for $\kappa > \sqrt{2}$ is larger than the thermodynamic critical field H_c. The latter is the critical field that is relevant for normal, or “type-I” superconductors. We now call H_{c2} the upper critical magnetic field. However, the material is not completely superconducting in the sense that the magnetic field vanishes everywhere in the material. Abrikosov found that a periodic distribution of the magnetic field, as a lattice, minimised the total energy. An experimentally observed Abrikosov lattice of this type is shown in Fig. 4.

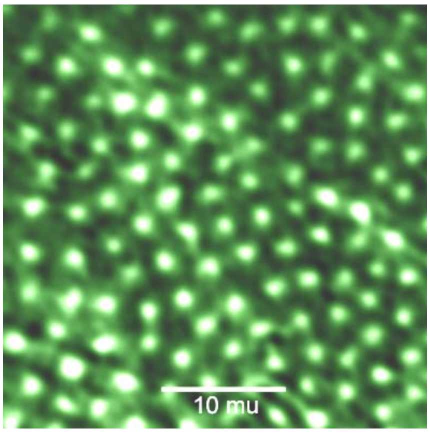

Figure 4. Abrikosov lattice of magnetic flux lines (vortices) in $NbSe_2$ – a type-II superconductor - visualised by magneto-optical imaging. The first pictures of such a vortex lattice were taken in 1967 by U. Essmann and H. Träuble, who sprinkled their sample surfaces with a ferromagnetic powder that arranges itself in a pattern reflecting the magnetic flux line structure.

The approach that worked for magnetic fields just below the upper critical field, where the order parameter is small and the nonlinear term in the first GL equation can be neglected, does not work for much weaker fields. However, by studying the nature of the solutions for fields just below H_{c2}, Abrikosov realised that they correspond to vortices in the order parameter and that this type of solution must be valid for weaker fields as well.

The point is that because we require the theory to be gauge invariant, the vector potential A and the phase φ of the order parameter $\Psi = |\Psi| \exp(i\varphi)$ appear in the combination $A - (\hbar/2e)\,\nabla\varphi$ in the first GL equation. Now, for the magnetic field to be constant inside the superconductor A has to grow. If, for example, we choose a gauge where the *y* component of *A* grows linearly in the *x* direction, so that $H_z = \partial A_y / \partial x$ with $A_y = H_z x$, the magnetic field points in the *z* direction. If the free energy is not to grow without limit, the growth in the vector potential has to be compensated by jumps in the phase. It turns out that this corresponds to vortex solutions in which the order parameter vanishes at the points of a regular (triangular or hexagonal) lattice and the phase of the order parameter changes by 2π on a closed contour around these lattice points.

Abrikosov discovered these solutions in 1953, but they were unexpected and he did not publish them until 1957. The suggestion by R.P. Feynman in 1955 that vortex filaments are formed in superfluid ^{4}He had then reached the Soviet Union. The level of scientific contact between East and West was very low during the Cold War and the work of Soviet scientists did not, in general, get much attention from researchers in the West. The work of Ginzburg-Landau was received with scepticism until L.P. Gorkov showed in 1959 that the GL equations could be derived from the microscopic BCS theory in the appropriate limit. Later, P.C. Hohenburg showed that the GL equations are valid not only close to the transition point in temperature or magnetic field but also at temperatures and in magnetic fields where the superconducting order is not small. The work of Abrikosov was not fully appreciated in the West until the 1960s, when superconductors with very high critical fields had been discovered.

Superfluid ^{3}He – a model anisotropic superfluid

We have already remarked that ^{3}He with its two electrons and three nucleons is a fermion. A large class of *interacting* fermion systems, like the normal electron liquid in many metals, can be described by Landau's fermi liquid theory developed during the 1950's. At the time of the BCS theory experimentalists had started to investigate the properties of liquid ^{3}He to see if it could be described by the Landau theory. J.C. Wheatly played a decisive role here by showing that liquid ^{3}He could indeed be very well described by Landau's fermi liquid theory below 100 mK. This is a much higher temperature than 2.7 mK, which later proved to be the critical temperature for a transition to the superfluid state. For a quantitative understanding of the liquid this result was important, since the atoms in liquid ^{3}He interacts strongly with each other.

Landau's theory is phenomenological and describes a system of interacting fermi particles in terms of "quasiparticles", a term he introduced. A quasiparticle can be viewed as a "bare" particle interacting with a cloud of surrounding particles. The theory has one parameter, the effective mass m^*, which describes the single-quasiparticle excitation spectrum, and a number of parameters that describe the effects of external fields. Often it is sufficient to have a few of these parameters, which can be determined from experiments. Landau's theory applies at "low enough" temperatures – a criterion that for liquid ^{3}He is very well satisfied at the transition temperature to superfluidity. In the mid 1960s Leggett was able to extend the Landau theory to the superfluid phases and calculate the (large) renormalisation of the nuclear spin susceptibility by interaction effects. His prediction agreed very well with later NMR measurements (see below).

Liquid ^{3}He was, as we have seen, of considerable experimental interest from the mid 1950s on. Only a few years after the publication of the BCS theory several authors – among them Pitaevskii; Brueckner, Soda, Anderson and Morel; and

Emery and Sessler – suggested that a BCS-like pair condensation into a superfluid state might occur in liquid ^{3}He. It was immediately clear that the strong repulsive interaction between the atoms would favour a relative orbital momentum state corresponding to *p*- or *d*-wave pairing in which the pair particles would be kept at some distance from each other. The superfluid would then be anisotropic, as we have discussed earlier in this text.

We now know that the condensed pairs of ^{3}He atoms are in a relative *p*-state (L=1), which means that the total wave function is antisymmetric with respect to an exchange of the spatial coordinates of two particles. Since the total wave function has to be antisymmetric (the Pauli principle) it follows that the wave function must be even with respect to an exchange of the spin coordinates of the two particles. The total spin of the pair must therefore be in a spin triplet state (S=1) with three possible values of the spin projection (S_z = +1, 0, -1) corresponding to the spin states (↑↑), (↑↓+↓↑)/√2 and (↓↓). Some properties of anistropic superfluids that can form under these circumstances were calculated theoretically. In 1961 P.W. Anderson and P. Morel proposed a superfluid condensate of pairs forming spin triplets with circular polarization (S_z=±1), where only the states (↑↑) and (↓↓) are involved (the ABM state). Two years later, however, R. Balian and N.R. Wertheimer and independently Y.A. Vdovin showed that lower energy is achieved with a pair state that also involves the spin state (↑↓+↓↑)/√2 (the BW state).

The experimental discovery of the superfluid A, B and A_1 phases in ^{3}He was made in 1972 by David Lee, Douglas Osheroff and Robert Richardson. Investigations, together with W.J. Gully, of the collective magnetic (*i.e.* spin-dependent) properties of the superfluid phases by nuclear magnetic resonance (NMR) were particularly useful in identifying the order parameter structure of these phases. In ordinary NMR experiments the system under study is subjected to a strong magnetic field H_0 in the z direction, which forces the spin S to precess around H_0. By applying a weak magnetic field H_{rf} of high frequency ω perpendicular to H_0, it is possible to induce transitions in S_z, the component along H_0, of magnitude $\pm\hbar$. This effect is observed as energy absorption from the magnetic field. If the spins do not interact, these transitions occur exactly when ω equals the Larmor frequency $\omega_L = \gamma H_0$, where γ is the gyromagnetic ratio of the nucleus. In fact, as long as the interactions in the system conserve spin it had been shown that the resonance remains at the Larmor frequency. On the other hand, for interactions that do not conserve spin, such as the spin-orbit interaction caused by the dipole coupling of the nuclear spins, a shift may occur. Normally this is expected to be very small, of the order of the line width. The NMR data published in connection with the experimental discovery of the superfluid phases was therefore a major surprise since it was found that although the resonance was still very sharp, it occurred at frequencies substantially higher than ω_L.

The solution to this puzzling fact was immediately found by Leggett, who showed that the NMR shifts are a consequence of the "spontaneously broken spin-orbit symmetry" of the spin-triplet *p*-wave state. As explained earlier, the meaning of this concept is that the preferred directions in spin and orbital space are long-range ordered, as illustrated for a simpler model in Fig. 1d and 1e. The tiny dipole interaction may take advantage of this situation; the *macroscopic quantum coherence* of the condensate raises the dipole coupling to macroscopic importance – the dipoles are aligned in the same direction and their moments add up coherently. In this way Leggett was first able to calculate the general NMR response of a spin-triplet *p*-wave condensate. In particular in the A-phase the transverse NMR frequency ω_t is given by

$$\omega_t^2 = \omega_L^2 + \Omega_A^2(T)$$

where $\Omega_A^2(T)$ is proportional to the dipole coupling constant and depends on temperature but not on H_0. Later, Leggett worked out the complete theory of the spin dynamics, whose predictions were experimentally confirmed in every detail. One of the predictions that were confirmed concerned "longitudinal" resonant NMR absorption in both the A and the B phase of energy from a high-frequency field oriented parallel with rather than perpendicular to the static field. In the A phase the resonant frequency of this longitudinal oscillation occurs at

$$\omega_l = \Omega_A(T)$$

where $\Omega_A(T)$ is the same frequency that appears in the expression for the transverse frequency.

Leggett identified the ABM state as a candidate to describe the A phase of superfluid ^{3}He, but noted that the BW state had been shown to have the lower energy. This, however, had only been proven within "weak-coupling" theory. After Leggett's prediction it became necessary to consider "strong-coupling" effects. The attractive interaction that is responsible for the pair formation in liquid helium is due to to the liquid itself, unlike (conventional) superconductors, where the pairing interaction between electrons is mediated by the lattice. P.W. Anderson and W. Brinkman showed that there is a conceptually simple effect that can explain the stabilisation of the ABM state over the BW state. It is based on a feedback mechanism: the pair correlations in the condensed state change the pairing interaction between the ^{3}He quasiparticles in a manner that depends on the state itself. As a specific interaction mechanism, Anderson and Brinkman considered spin fluctuations and found that a stabilisation of the state first proposed by Anderson and Morel is possible (hence the initials of all three authors are used to describe this state – the ABM state). This only happens at somewhat elevated pressures, when the spin fluctuations become more pronounced. This left room for the B phase to be identified with the BW state, which was soon done. Finally, V. Ambegaokar and N.D. Mermin identified the A1 phase, which appears at higher magnetic fields, with a state where only one of the spin states (↑↑) and (↓↓) is involved.

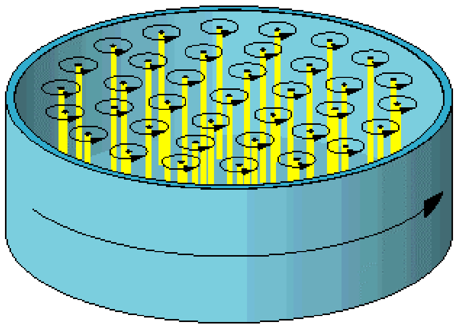

Figure 5. Vortex lines in a superfluid are analogous to the flux lines that occur in a type-II superconductor when it is placed in a magnetic field (Cf. Fig. 4). The picture illlustrates vortex lines in rotating superfluid ^{3}He, where the vortex structure is particularily rich. The vortex lines are shown in yellow, and the circulating flow around them is indicated by arrows.

Importance

The Ginzburg-Landau (GL) theory has been important in many fields of physics, including particle physics, where it is used in string theory. Today, the GL theory is extensively used to describe superconductive properties that are important in practical applications. This theory is able to describe, for example, spatially varying superconducting order, superconductivity in strong magnetic fields and fluctuating – time-dependent – superconducting order.

Abrikosov's theory of superconductors in a magnetic field created a new field of physics – the study of type-II superconductors. After the discovery in 1986 of the ceramic "high-temperature" superconductors, which are extreme type-II superconductors, by Gerd Bednorz and Alex Müller (Nobel Prize 1987) research to understand and use these new materials has become a very large activity. The vortex/flux lines discovered by Abrikosov are very important for the properties of these materials – the term "vortex matter" is used.

The work of Leggett was crucial for understanding the order parameter structure in the superfluid phases of ^{3}He. His discovery that several simultaneously broken symmetries can appear in condensed matter is, however, of more general importance for understanding complex phase transitions in other fields as well, like liquid crystal physics, particle physics and cosmology.

Further reading

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