100 Years of Light Quanta

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Max Planck: October 19, 1900

Interpolation formula for thermal radiation distribution – a brilliant success

December 14, 1900:

Model: Ensemble of 1-dimensional charged harmonic oscillators exchanging energy with radiation field

– reached "correct" equilibrium distribution only if oscillator energy states were discrete

$$E_n = nh\nu$$

Albert Einstein: 1905

Found two suggestions that **light is quantized**

- Structure of Planck's entropy for high frequencies

- The photoelectric effect

He noted in later studies –

- Momentum of Quantum (1909) $$\frac{h\nu}{c}$$

- New Derivation of Planck's law (1916)

A = Spontaneous radiation probability

B = Induced radiation rate

Compton effect: 1923

Completed picture of **particle-like behavior of quanta** - soon known as **photons** (1926)

L. de Broglie, W. Heisenberg, E. Schrödinger: 1924-26

Quantum mechanics - told all about atoms

But radiation theory was still **semi-classical** until P. Dirac devised **Quantum Electrodynamics** in 1927

Split real field into two complex conjugate terms

$$E = E^{(+)} + E^{(-)}$$

$$E^{(-)} = \left(E^{(+)}\right)^*$$

$E^{(+)}$ contains only positive frequencies $\sim e^{-i\omega t}$

$E^{(-)}$ contains only negative frequencies $\sim e^{i\omega t}$

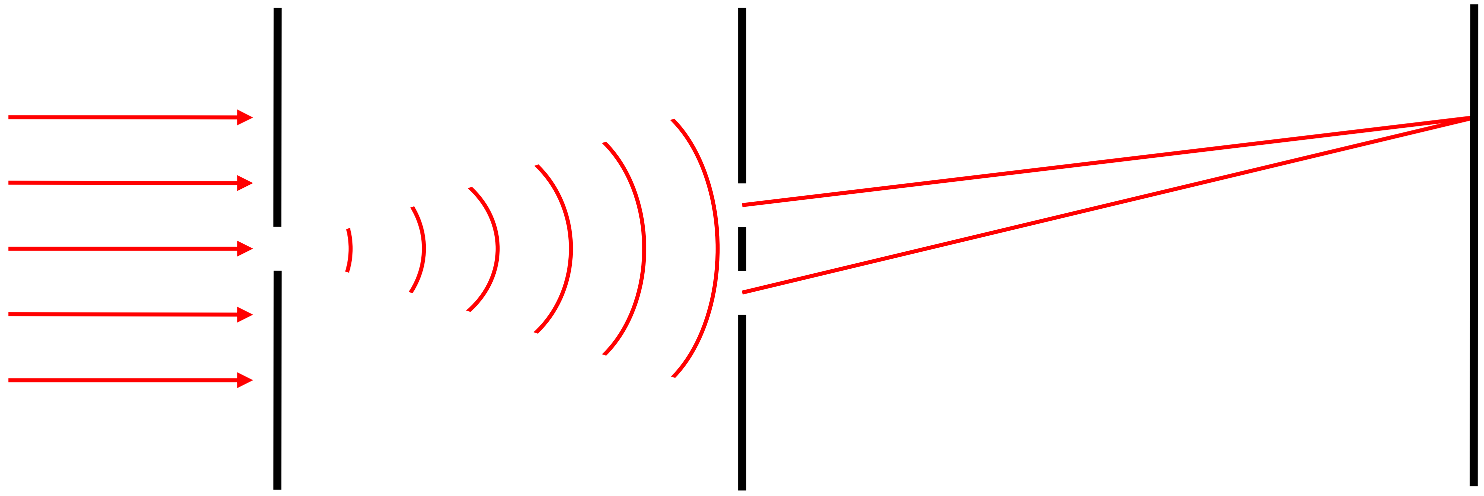
$E^{(\pm)}$ physically equivalent (**classically**)

Define correlation function

$$G^{(1)}(r_1 t_1 r_2 t_2) = \langle E^{(-)}(r_1 t_1) E^{(+)}(r_2 t_2) \rangle$$

Young's 2-pinhole experiment measures:

$$G^{(1)}(\mathrm{r}_1\mathrm{t}_1\mathrm{r}_1\mathrm{t}_1) + G^{(1)}(\mathrm{r}_2\mathrm{t}_2\mathrm{r}_2\mathrm{t}_2) + G^{(1)}(\mathrm{r}_1\mathrm{t}_1\mathrm{r}_2\mathrm{t}_2) + G^{(1)}(\mathrm{r}_2\mathrm{t}_2\mathrm{r}_1\mathrm{t}_1)$$



Coherence maximizes fringe contrast

Let $x = (r,t)$

Schwarz Inequality:

$$\left|G^{(1)}(x_1 x_2)\right|^2 \leq G^{(1)}(x_1 x_1) G^{(1)}(x_2 x_2)$$

Optical coherence:

$$\left|G^{(1)}(x_1 x_2)\right|^2 = G^{(1)}(x_1 x_1) G^{(1)}(x_2 x_2)$$

Sufficient condition: $G^{(1)}$ factorizes

i.e. $G^{(1)}(\mathrm{x}_1 \mathrm{x}_2) = \mathcal{E}^*(\mathrm{x}_1)\, \mathcal{E}(\mathrm{x}_2)$

~ also necessary:

Titulaer & *G. Phys. Rev.* 140 (1965), 145 (1966)

Quantum Theory:

$E^{(\pm)}$ Are operators on quantum state vectors

$E^{(+)}(rt)$ Annihilation operator, lowers n

$$|n\rangle \rightarrow |n-1\rangle$$

$E^{(-)}(rt)$ Creation operator, raises n

$$|n\rangle \rightarrow |n+1\rangle$$

Lowering n stops with $n = 0$, vac. state

$$E^{(+)}(rt)|vac.\rangle = 0$$

Ideal photon counter:

- point-like, uniform sensitivity

Transition Amplitude $\langle f | E^{(+)}(rt) | i \rangle$

- Square, sum over $| f \rangle$
- Use completeness of $| f \rangle$

$$\sum_f | f \rangle\langle f | = 1 \text{ (unit op.)}$$

Total transition probability ~

$$\sum_f \left| \langle f | E^{(+)}(rt) | i \rangle \right|^2 = \sum_f \langle i | E^{(-)}(rt) | f \rangle\langle f | E^{(+)}(rt) | i \rangle$$

$$= \langle i | E^{(-)}(rt) E^{(+)}(rt) | i \rangle$$

Initial states $\left| i \right\rangle$ random

Take ensemble average over $\left| i \right\rangle$

Density Operator: $\rho = \{ | i \rangle \langle i | \}_{Average}$

~ Then **averaged counting probability** is

$$\{ \langle i | E^{(-)}(rt) E^{(+)}(rt) | i \rangle \}_{Av.} = Trace\{ \rho E^{(-)}(rt) E^{(+)}(rt) \}$$

To discuss coherence we define

$$G^{(1)}(r_1 t_1 r_2 t_2) = Trace\{\rho E^{(-)}(r_1 t_1) E^{(+)}(r_2 t_2)\}$$

- obey same Schwarz Inequality as classical $G^{(1)}$

Upper bound attained likewise by factorization,

$$G^{(1)}(r_1 t_1 r_2 t_2) = \mathcal{E}^*(r_1 t_1)\, \mathcal{E}(r_2 t_2)$$

Statistically steady fields: $G^{(1)} = G^{(1)}(t_1 - t_2)$

- If optically coherent,

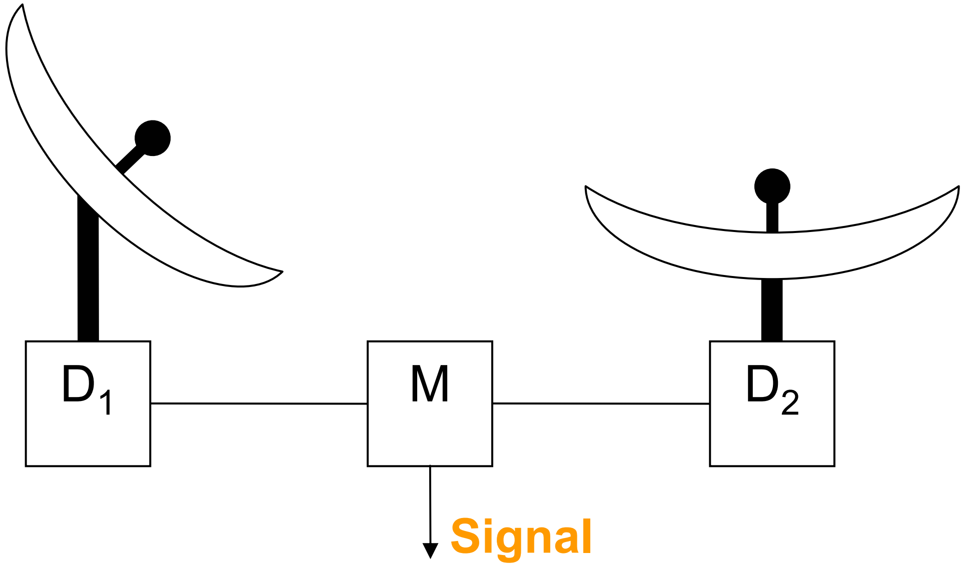
$$G^{(1)}(t_1 - t_2) = \mathcal{E}^*(t_1)\, \mathcal{E}(t_2)$$

The only possibility is: $\mathcal{E}(t) \sim e^{-i\omega t}$ for $\omega > 0$

R. Hanbury Brown and R. Q. Twiss

Intensity interferometry

Two square-law detectors



Ordinary (Amplitude) interferometry measures

$$G^{(1)}(rtr't') \equiv \left\langle E^{(-)}(rt)E^{(+)}(r't') \right\rangle_{Ave.}$$

Intensity interferometry measures

$$G^{(1)}(rtr't'r't'rt) = \left\langle E^{(-)}(rt)E^{(-)}(r't')E^{(+)}(r't')E^{(+)}(rt) \right\rangle$$

The two photon dilemma!

THE PRINCIPLES OF QUANTUM MECHANICS

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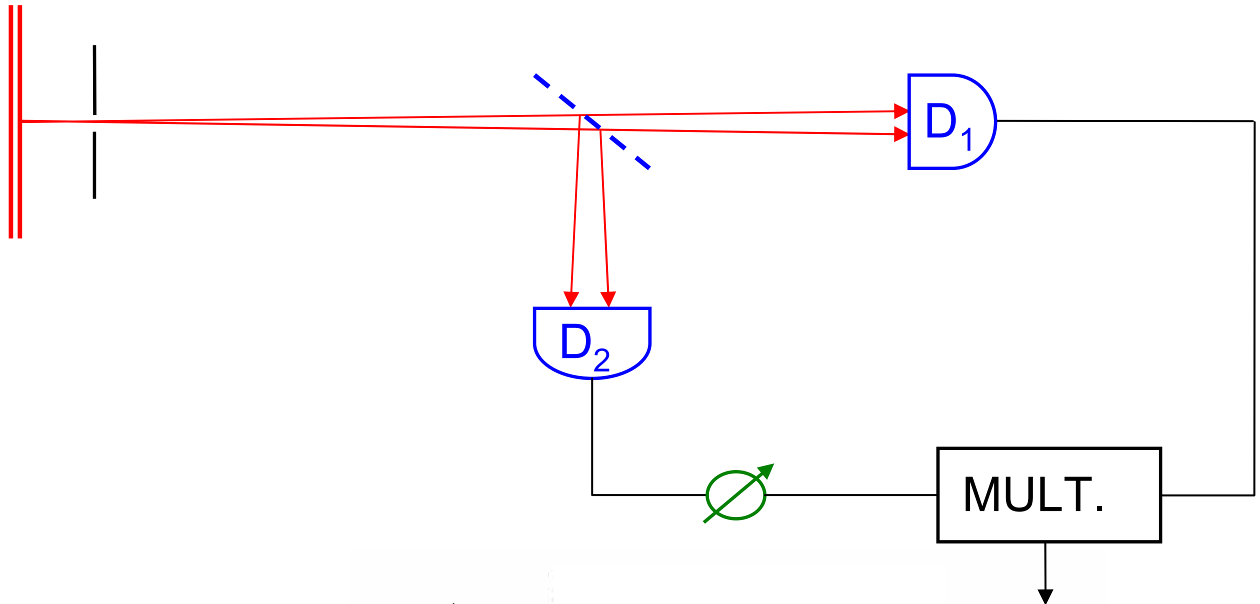
THIRD EDITION

§3, P.9 ...

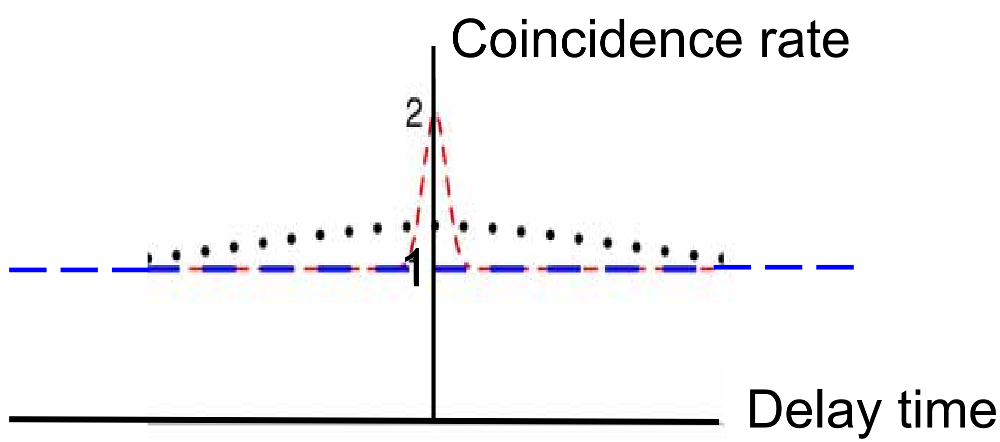
Some time before the discovery of quantum mechanics people realized that the connexion between light waves and photons must be of a statistical character. What they did not clearly realize, however, was that the wave function gives information about the probability of *one* photon being in a particular place and not the probable number of photons in that place. The importance of the distinction can be made clear in the following way. Suppose we have a beam of light consisting of a large number of photons split up into two components of equal intensity. On the assumption that the intensity of a beam is connected with the probable number of photons in it, we should have half the total number of photons going into each component. If the two components are now made to interfere, we should require a photon in one component to be able to interfere with one in the other. Sometimes these two photons would have to annihilate one another and other times they would have to produce four photons. This would contradict the conservation of energy. The new theory, which connects the wave function with probabilities for one photon, gets over the difficulty by making each photon go partly into each of the two components. Each photon then interferes only with itself. Interference between two different photons never occurs.

The association of particles with waves discussed above is not restricted to the case of light, but is, according to modern theory, of universal applicability. All kinds of particles are associated with waves in this way and conversely all wave motion is associated with

Hanbury Brown and Twiss ‘56



Pound and Rebka ‘57



Define **higher order coherence** (*e.g.* second order)

$$G^{(2)}(x_1, x_2, x_3, x_4) = \langle E^{(-)}(x_1)\, E^{(-)}(x_2)\, E^{(+)}(x_3)\, E^{(+)}(x_4)\rangle$$
$$= \mathcal{E}^*(x_1)\, \mathcal{E}^*(x_2)\, \mathcal{E}(x_3)\, \mathcal{E}(x_4)$$

→ Joint count rate factorizes

$$G^{(2)}(x_1, x_2, x_2, x_1) = |\mathcal{E}(x_1)|^2 |\mathcal{E}(x_2)|^2$$

→ Wipes out HB-T correlation

n^{th} order coherence, $n \rightarrow \infty$

What field states factorize all $G^{(n)}$?

Recall normal ordering

- Sufficient to have: $E^{(+)}(rt)\left|\ \right\rangle = \mathcal{E}(rt)\left|\ \right\rangle$

~ defines **coherent states**

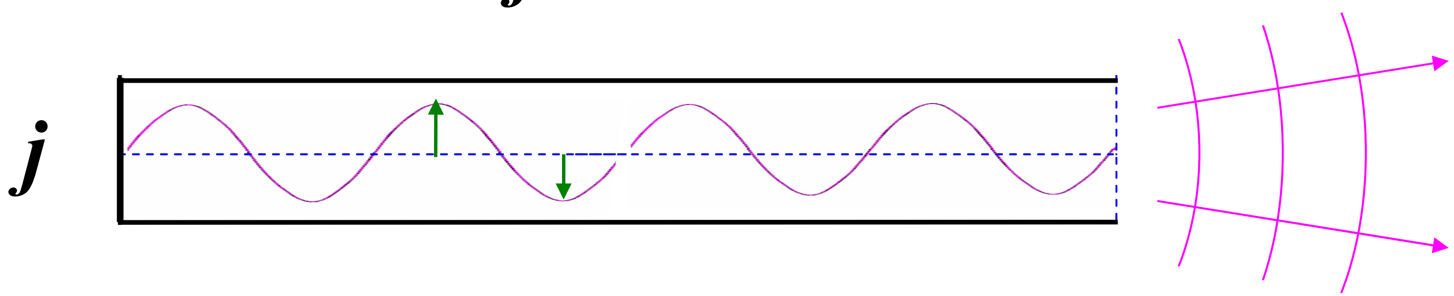
Convenient basis for averaging normally ordered products

All $G^{(n)}$ can factorize → **Full coherence**

Any classical (i.e., predetermined) current $\boldsymbol{j}$ radiates coherent states

~ R.G. *Phys. Rev.* 84, ’51

What is current $\boldsymbol{j}$ *for a laser?*



Strong oscillating polarization current $\boldsymbol{j} = \dfrac{\partial \boldsymbol{P}}{\partial t}$

Quantum Optics = Photon Statistics

Quantum Field Theory – for bosons

Field oscillation modes ↔ harmonic oscillators

For harmonic oscillator:

a lowers excitation $a\,|\,n\rangle = \sqrt{n}\,|\,n - 1\rangle$

$a^{\dagger}$ raises excitation $a^{\dagger}\,|\,n\rangle = \sqrt{n+1}\,|\,n+1\rangle$

$$aa^{\dagger} - a^{\dagger}a = 1$$

Special states: $a|\alpha\rangle = \alpha|\alpha\rangle$

α = any complex number

$$|\alpha\rangle = e^{-\frac{1}{2}|\alpha|^2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle$$

$$P(n) = \frac{|\alpha|^{2n}}{n!} e^{-|\alpha|^2}$$, Poisson distribution

$$\langle n\rangle = |\alpha|^2$$

~ single mode coherent states

Superposition of coherent excitations:

Source #1 → $|\alpha_1\rangle$

Source #2 → $|\alpha_2\rangle$

Sources #1 and #2 → $e^{\frac{1}{2}(\alpha_1^* \alpha_2 - \alpha_1 \alpha_2^*)} |\alpha_1 + \alpha_2\rangle$

Combined density operator: $\rho = |\alpha_1 + \alpha_2\rangle\langle\alpha_1 + \alpha_2|$

With n sources $\rho = |\alpha\rangle\langle\alpha|$, $\alpha = \sum_{\mathrm{j}=1}^{\mathrm{n}} \alpha_{\mathrm{j}}$

For $n \rightarrow \infty$, α_j's random

Sum α has a random-walk probability distribution – Gaussian

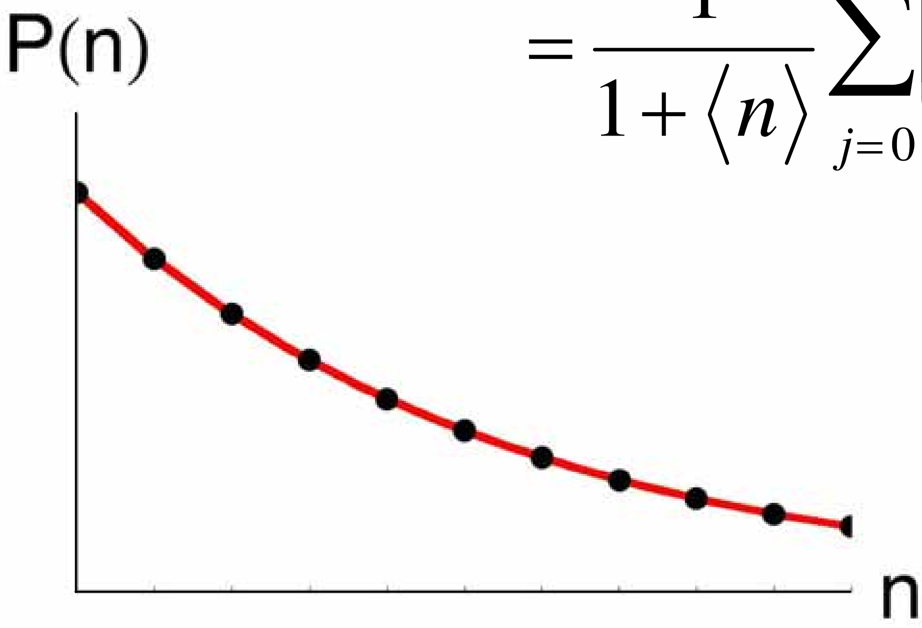
$$P(\alpha) = \frac{1}{\pi\left\langle\left|\alpha^2\right|\right\rangle} e^{\frac{-|\alpha|^2}{\left\langle|\alpha|^2\right\rangle}}$$

But $\left\langle|\alpha|^2\right\rangle_{AV.} = \langle n\rangle$, mean quantum number

e.g. Gaussian distribution of amplitudes $\{\alpha_n\}$
Single-mode density operator:

$$\rho_{chaotic} = \frac{1}{\pi\langle n\rangle}\int e^{-\frac{|\alpha|^2}{\langle n\rangle}}|\alpha\rangle\langle\alpha|d^2\alpha$$

$$= \frac{1}{1+\langle n\rangle}\sum_{j=0}^{\infty}\left(\frac{\langle n\rangle}{1+\langle n\rangle}\right)^j |j\rangle\langle j|$$



Two-fold joint count rate:

$$G^{(2)}(x_1 x_2 x_2 x_1) = G^{(1)}(x_1 x_1) G^{(1)}(x_2 x_2) + \underbrace{G^{(1)}(x_1 x_2) G^{(1)}(x_2 x_1)}_{\text{HB-T Effect}}$$

Note for $x_2 \rightarrow x_1$:

$$G^{(2)}(x_1 x_1 x_1 x_1) = 2\,[G^{(1)}(x_1 x_1)]^2$$

If the density operator for a single mode can be written as:

$$\rho = \int P(\alpha) |\alpha\rangle\langle\alpha| d^2\alpha$$

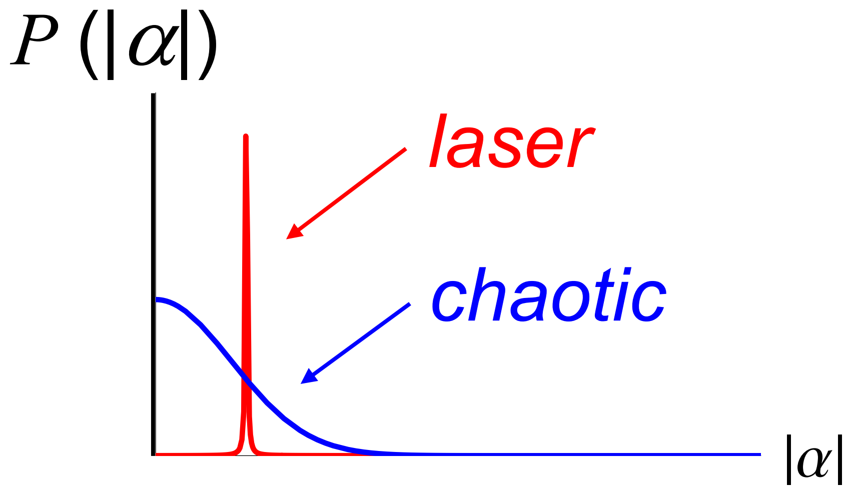
Then $\langle a^{\dagger n} a^m \rangle = Tr\,(\rho a^{\dagger} a^m) = \int P(\alpha) \alpha^{*\mathrm{n}}\, \alpha^m d^2\, \alpha$

Operator averages become integrals

$P(\alpha)$ = quasi-probability density

Scheme works well for pseudo-classical fields, but is not applicable to some classes of fields *e.g.* “squeezed” fields, (no P-function exists).

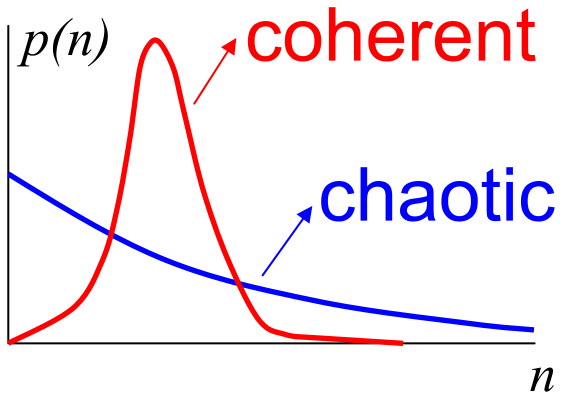
One mode excitation:



Photocount distributions (w = average count rate)

Coherent state: $$p(n) = \frac{(wt)^n}{n!} e^{-wt}$$

Chaotic state: $$p(n) = \frac{(wt)^n}{(1+wt)^{n+1}}$$



Distribution of time intervals until first count:

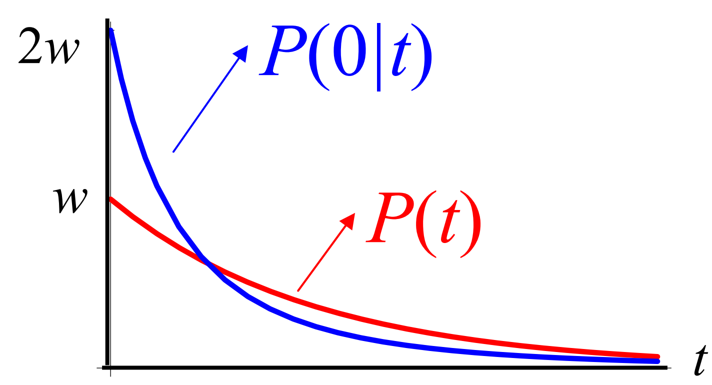
Coherent: $P(t) = we^{-wt}$

Chaotic: $P(t) = \dfrac{w}{(1+wt)^2}$

Given count at t = 0, distribution of intervals until next count:

Coherent: $P(0\,|\,t) = we^{-wt}$

Chaotic: $P(0\,|\,t) = \dfrac{2w}{(1+wt)^3}$



Quasi-probability representations for quantum state ρ

Define characteristic functions:

$$\chi(\lambda, s) = Trace\{\rho e^{\lambda a^\dagger - \lambda a}\} e^{\frac{s}{2}\|\lambda|^2}$$

Family of quasi-probability densities:

$$W(\alpha, s) = \frac{1}{\pi}\int e^{\alpha\lambda^* - \alpha^*\lambda}\chi(\lambda, s) d^2\lambda$$

$s = 1$ $\quad \mathrm{W}(\alpha,1) = \mathrm{P}(\alpha)$ P-rep.

$s = 0$ $\quad \mathrm{W}(\alpha,0) = \mathrm{w}(\alpha)$ Wigner fn.

$s = -1$ $\quad \mathrm{W}(\alpha,-1) = \frac{1}{\pi}\langle\alpha|\rho|\alpha\rangle$ Q-rep.

Later Developments:

- Measurements of photocount distributions
  ~ Arecchi, Pike, Bertolotti…
- Photon anti-correlations ~ Kimble, Mandel
- Quantum amplifiers
- Detailed laser theory ~ Scully, Haken, Lax
- Parametric down-conversion – entangled photon pairs
- Application to other bosons
    - bosonic atoms (BEC)
    - H.E. pion showers
    - HB-T correlations for He* atoms
- Statistics of Fermion fields ~ with K.E. Cahill

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